

# Kansas Multi-Tier System of Supports

## ● Structuring Guide: Module 2 Mathematics

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## Introduction to Document

The *Kansas Multi-Tier System of Supports: Structuring Guide* has been created to assist schools in creating the structures necessary to begin the implementation of a Multi-Tier System of Supports (MTSS). This document serves as a workbook for either schools working with Recognized MTSS Trainers (a current list can be found at [www.kansasmtss.org](http://www.kansasmtss.org)) or as a do-it-yourself guide for schools taking on the challenge themselves. This document provides an explanation of why each component is important as well as suggests steps that have helped other schools successfully complete the tasks and decision making necessary for creating structures that support a sustainable system. Content area specific documents for reading, mathematics, and behavior are companion documents to this one, providing information specific to each content area. All Kansas MTSS documents are aligned with the *Kansas Multi-Tier System of Supports: Innovation Configuration Matrix (ICM)*, which describes the critical components of a MTSS and what each looks like when fully implemented, and the *Kansas Multi-Tier System of Supports: Research Base*, which provides a basic overview of the research support for a MTSS.

## Acknowledgements

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## Introduction

Multi-Tier  
Framework for  
Math: Research-  
Based Principles &  
Practices

Although research in math is not as complete as it is for reading, the MTSS model for mathematics is built on the best evidence available at this time. The most recent research and recommendations of math experts have been combined to create the current model. Some pieces of the model come from the Center on Instruction ([www.centeroninstruction.org](http://www.centeroninstruction.org)), which summarizes nine key research studies and identifies overarching principles for an effective multi-tier framework for school-age children in mathematics. These **principles** are:

1. Increased instructional time and supports.
2. Small-group instruction.
3. Explicit methods of instruction.
4. The use of concrete and pictorial representations.
5. Strategy instruction for problem solving.
6. A focus on basic facts and word problems.
7. Alignment of Tier 2 instruction with Tier 1 to maximize screening and progress monitoring to focus instruction on deficit areas.  
(Newman-Gonchar, Clarke, & Gersten et al., 2009b)

*Principals need to be familiar with these principles and practices and what they look like in the classroom.*

Other key pieces of the model come from another similar meta-analysis of the research. From this research, this is hardly a new concept. What Works Clearinghouse developed a multi-tier model of intervention that includes these **practices**:

1. Screen all students.
2. Focus instruction on whole numbers for grades K-5 and rational numbers for grades 4-8.
3. Provide systematic and explicit instruction.
4. Teach problem-solving structures.
5. Provide visual representations.
6. Build fluency with basic arithmetic facts.
7. Provide progress monitoring.
8. Include motivational strategies.  
(Gersten et al., 2009a)

These principles and eight practices, along with other information taken from the research base, were utilized in building the structures for a Multi-Tier System of Supports (MTSS) for mathematics. It is important that the leadership team and building staff begin the MTSS structuring for mathematics with an understanding of these principles and practices, as well as the following critical concepts.

### Critical Concepts for MTSS Math

The principal of one MTSS secondary field test school met with his staff of 75 just before beginning implementation. He asked, “How many of you have said, ‘I can’t do math?’” Approximately 10 staff

members raised their hands. Then he asked, “How many of you have said, ‘I can’t read?’” No one raised a hand. Then he asked, “Why is it that we as adults think it’s okay to not be able to do math, but we have much higher expectations for reading? Are we communicating that attitude to our students? From now on, all of us, no matter what we teach, will communicate our expectation that everyone in our building will learn to read and perform math on grade level. And we’ll all work to make that happen.”

A discussion of critical concepts in math is important for ensuring that personal experiences and feelings about math do not interfere with the ability to hold high expectations for student achievement in math. One example of this is the research and discussion about differences in expectations for boys and girls regarding math achievement (National Mathematics Advisory Panel, 2008). Teacher expectations for student achievement and other beliefs about math can be communicated to students in very subtle ways. A discussion of critical concepts in math can help ensure that teachers understand some of the basic issues in the field and provide mutual support for developing productive practices for student learning and math instruction.

The following five critical concepts for math for school-age students was adapted from Riccomini and Witzel (2010), but the source for each critical concept is also referenced.

1. All students can be mathematically proficient (Kilpatrick, Swafford, & Findell, 2001).
2. All students need a high quality mathematics program (National Mathematics Advisory Panel, 2008).
3. Effective mathematics programs must teach conceptual understanding, computational fluency, factual knowledge, and problem-solving skills (National Mathematics Advisory Panel, 2008).
4. Effective mathematics instruction matters and it significantly impacts student learning (National Mathematics Advisory Panel, 2008; Newman-Gonchar, Clarke, & Gersten et al., 2009b).
5. Teachers should use a balance of “student-centered” and “teacher-centered” instruction in the core mathematics program (National Mathematics Advisory Panel, 2008). In student-centered instruction, the teacher guides students in constructing meaning through discovery learning. In teacher-centered instruction, the teacher directly teaches important information using explicit instruction. Teacher-centered instruction has been shown to be especially important for students who are struggling, low achievers, and students with disabilities (Gersten et al., 2009b).



## MATH CRITICAL CONCEPTS ACTIVITY- GRADES K-12

Instructions: As a group, discuss each statement and complete the chart. As you discuss these statements, think about ALL STUDENTS and ALL SUBGROUPS reflected in your building.

STATEMENT	Agree	Why	Evidence that the building supports this statement	Disagree	Why	Practices that would need to change or be implemented to support this statement
All students can be mathematically proficient.						
All students need a high-quality mathematics program.						
Effective mathematics programs must teach conceptual understanding, computational fluency, factual knowledge, and problem solving.						
Effective mathematics instruction matters and significantly impacts student learning.						
Teachers will use a balance of “student-centered” and “teacher-centered” instruction in the core mathematics program.						

The following is a similar list of guiding principles for math for preschool children:

1. Young children have the ability to learn mathematics, although this potential is often not realized in the preschool years (Committee on Early Childhood Mathematics, National Research Council, 2009). Preschool provides an opportunity to expand children's knowledge and foster a lifelong enjoyment of math (National Institute for Early Education Research, 2010).
2. High-quality mathematical programs should be provided in early childhood programs (Committee on Early Childhood Mathematics, National Research Council, 2009).
3. Experience in high-quality mathematics instruction is especially important for preschool children living in poverty (Committee on Early Childhood Mathematics, National Research Council, 2009).
4. Preschool teachers should intentionally introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies (National Association for the Education of Young Children and the National Council of Teachers of Mathematics, 2009; Committee on Early Childhood Mathematics, National Research Council, 2009).

The focus on mathematical experiences in preschool should be on:

- a. Numbers (including whole number, operations, and relations).
- b. Geometry, spatial relations, and measurements (Committee on Early Childhood Mathematics, National Research Council, 2009).

The preschool core math curriculum supports the acquisition of knowledge in skills in both areas; however, a significant amount of focus and time should be devoted to numbers with secondary focus on geometry, spatial relations, and measurement (Committee on Early Childhood Mathematics, National Research Council, 2009).

### **The Focus of Math MTSS in Preschool**

Researchers have yet to validate specific assessment and intervention practices or provide concrete guidance related to the implementation of tiered support in the area of mathematics for very young children (B. Clarke, in personal communication, November 27, 2012). While some have identified promising practices, such as the use of Preschool Numeracy Indicators (sometimes referred to as Early Numeracy Skill Indicators) as a method for universal screening (VanDerHeyden, Broussard, & Cooley, 2006) (Methe, Hintze, & Floyd, 2008) (Hojnoski, Silberglitt, & Floyd, 2009), and identified positive effects of providing evidence based early childhood core math curricula (Clements D. H., 2011), specific interventions for Tier 2 or Tier 3 support have yet to be established (V. Buysse in personal communication, April 4, 2012). Although many of the practices related to the implementation of math

MTSS in elementary and secondary systems hold promise for use in preschool programs, the sequence of activities and practices for preschool is somewhat different. Perhaps the most significant challenge to the implementation of an effective MTSS in preschool is the fact that often little if any formal mathematical instruction is provided, evidence based math curriculum is rarely implemented, and early childhood professionals are often lacking in the knowledge necessary to provide appropriate instruction in this content area (Ginsburg, Lee, & Boyd, 2008).

For these reasons, the initial work of the leadership team is to identify and implement an evidenced based preschool math curriculum. Successful implementation of such a curriculum will hinge on appropriate professional development and other support such as instructional coaching provided to early childhood staff identified to carry out this work. As stated previously, early childhood staff may have little knowledge or experience in this content area. Therefore, the cognitive demands placed on preschool staff to learn and implement a new curriculum may hinder their ability to put universal screening practices into place at the same time as their elementary colleagues. However, it will be necessary for the leadership team to identify assessment tools or practices appropriate for monitoring individual and group progress in the preschool core curriculum for appropriate instructional planning. Such assessments are often linked directly to the curriculum (curriculum based assessments) and are provided as a part of the overall curriculum by the publishers.

Once curriculum and assessment practices are implemented appropriately, steps can be taken to begin implementing preschool universal screening assessments.

## MATH CRITICAL CONCEPTS ACTIVITY - PRESCHOOL

Instructions: As a group, discuss each statement and complete the chart. As you discuss these statements, think about ALL STUDENTS and ALL SUBGROUPS reflected in your building.

STATEMENT	Agree	Why	Evidence that the building supports this statement	Disagree	Why	Practices that would need to change or be implemented to support this statement
Young children have the ability to learn mathematics, although this potential is often not realized in the preschool years.						
High-quality mathematical programs should be provided in early childhood programs.						
The focus of mathematical experiences in preschool should be on early numeracy (including whole number, operations, and relations), geometry, spatial relations, and measurements.						
Experience in high-quality mathematics instruction is especially important for preschool children living in poverty.						
Preschool teachers should actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies						

These lists serve as a starting point for discussion, and the building leadership team may add to or change the list as these critical concepts about math are examined.



### **Mathematical Proficiency**

In the book *Adding it Up: Helping Children Learn Mathematics* (National Research Council, 2001, p. 5), mathematical proficiency is the terminology used to convey the development of success in mathematics. The report states that mathematical proficiency has five strands and “the most important feature of mathematical proficiency is that these five strands are interwoven and interdependent.” The five intertwined strands of mathematical proficiency are:



- Understanding mathematics: Comprehending mathematical concepts, operations, and relations—knowing what mathematical symbols, diagrams, and procedures mean.
- Computing fluently: Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
- Applying concepts to solve problems: Being able to formulate problems mathematically and devise strategies for solving them using concepts and procedures appropriately.
- Reasoning logically: Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.
- Engaging with mathematics: Seeing mathematics as sensible, useful, and doable—if you work at it—and being willing to do the work.

This is hardly a new concept. In his 1956 article, “Meaning and Skill: Maintaining the Balance,” William A. Brownell (as cited in Boerst & Schielack, 2003) stated that teachers must teach so that students acquire both conceptual understanding (meaning) and skills as evidenced in computational fluency. Implications of this concept are summed up by Wu when he states “in mathematics, skills and understanding are completely intertwined. In most cases, the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding. There is not ‘conceptual understanding’ and ‘problem-solving skill’ on the one hand and ‘basic skills’ on the other. Nor can one acquire the former without the latter” (Wu, 1999, p.1).

The National Mathematics Advisory Panel (2008) found that lack of proficiency, particularly in fractions, is a major obstacle for most students. It is essential that students have conceptual and procedural knowledge of fractions and proportional reasoning and have the ability to apply this knowledge toward accurate problem solving. The IES Practice Guide on fractions

(<http://ies.ed.gov/ncee/www/PracticeGuide.aspx?sid=15>) includes five recommendations for improving students' proficiency with fractions:

1. Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.
2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
3. Help students understand why procedures for computations with fractions make sense.
4. Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.
5. Professional development programs should place a high priority on improving teachers' understanding of fractions and how to teach them. (Siegler et al., 2010)

Each recommendation in the IES Practice Guide on fractions has information and suggestions regarding how to carry out the recommendation as well as potential roadblocks and solutions.

### Models for K-12 Math Intervention

In Kansas, two approaches have historically been used for math interventions. One approach focuses on intervention through a diagnostic process to identify concept/skill deficits and establish intervention groups based on building mathematics proficiencies (Gersten et al., 2009b). The other approach designs interventions based on state assessment-tested indicators. The MTSS framework for mathematics uses the mathematics proficiencies approach. It is critical that teams discuss and understand this approach for providing supplemental and intensive interventions to students.

The **mathematics proficiencies** approach utilizes a diagnostic assessment process to create small, homogenous groupings for supplemental and intensive instruction. Mathematics proficiencies may be skills/concepts taught at previous grade levels that were never mastered by the students. The **tested indicators** approach prioritizes the subject matter on the state assessment and arranges additional small-group instruction for students identified as being below proficient on those indicators. The latter approach does not plan for remediation of missing prerequisite skills/concepts. The tested indicator approach does not address skills/concepts below grade level. The mathematics proficiencies approach for intervention groups almost always addresses skills/concepts that are below grade level because groups are designed to teach students where the deficits begin, regardless of the grade level of the deficit.

Grouping  
Practices for K-12  
Math  
Intervention  
Groups

The MTSS  
Mathematical  
Proficiencies  
Model

Problems with  
the Tested  
Indicators Model

Most state assessments fail to produce the kinds of data that will improve teaching and learning (Popham, 2003). Although standardized achievement tests can help identify broad areas of strength and weakness, they have only limited value for diagnostic purposes. Another problem with the tested indicator approach is that missing skills/concepts from lower grades are not identified and remediated using a planned protocol whereas addressing this shortcoming is a critical component of the MTSS hybrid model. A further problem is that intervention groups in the tested indicator approach often are organized to address weaknesses in assessed state indicators, and this emphasis sometimes leads to a lack of instruction in nonassessed indicators. This lack of attention to nonassessed indicators can result in poor math outcomes for students in later grades if the foundational prerequisite skills/concepts for higher level math functions were never mastered.

No conclusive research in math exists to support the use of one approach over the other. The MTSS framework for mathematics uses the mathematics proficiencies approach because it aligns with the MTSS hybrid model as well as our understanding of the importance of foundational skill knowledge for later success. Understanding of, and agreement with, the mathematics proficiencies approach of intervention in math is important because all materials used for MTSS Math Implementation training are based on this approach.

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## Assessment

Universal  
Screening

The overall purpose of mathematics assessment must be to improve student learning. Assessment needs to support the learning of important mathematics and furnish useful information to both teachers and students. The National Council of Teachers of Mathematics (NCTM) maintains that assessment should be an integral part of instruction, providing not only the teacher, but also the student with information about the student's learning.

*Principals need to understand the purpose of the universal screening tool and the usefulness of its data.*

### Selecting a Universal Screening Assessment

Universal screening assessments must be reliable, valid, and efficient. Specific recommendations for criteria for these features can be found in the Institute for Educational Services Math RtI Practice Guide (Gersten et al., 2009a). The IES Math RtI Guide recommends that a universal screening assessment take less than 20 minutes to administer. Universal screening in MTSS addresses basic critical skills/concepts, not every concept taught in the classroom.

Universal screening tools are reviewed and validated by the National Center on Response to Intervention (NCRtI)

<http://rti4success.org/screeningTools>, and the Best Evidence Encyclopedia – Elementary Guide:

[http://www.bestevidence.org/word/math\\_Jan\\_05\\_2010\\_guide.pdf](http://www.bestevidence.org/word/math_Jan_05_2010_guide.pdf) or Middle School/High School Guide:

[http://www.bestevidence.org/word/mhs\\_math\\_Mar\\_11\\_2009\\_su\\_m.pdf](http://www.bestevidence.org/word/mhs_math_Mar_11_2009_su_m.pdf).

All Kansas MTSS Implementation materials are written based on the use of data that measure the identified critical skills and concepts at each grade level. Without data for the identified critical skills and concepts at each grade level, leadership and collaborative teams will not be able to utilize the MTSS Implementation materials.

Universal  
Screening for  
Preschool

### Universal Screening for Preschool

Early numeracy skills at kindergarten entry appear to be a strong predictor of later school success in both reading and mathematics (Duncan, Dowsett, Claessens, Magnuson, Huston, & Klebanov, 2007) suggesting that mathematical competency should be a primary instructional target at the preschool level. For this reason, leadership teams may find it beneficial to integrate preschool programs into their MTSS activities, including universal screening.

Mathematical  
Tasks for  
Universal  
Screening at  
Preschool

### Universal Screening for Preschool

Once a core curriculum for preschool has been identified and implemented with fidelity, the leadership team may take steps to integrate preschool into building-wide universal screening activities. Screening has been shown to accurately identify students who are likely to have future math difficulties, indicating that even at these early ages, math proficiency is multi-faceted (Morgan, Farkas, &

Qiong, 2009). Preschool screening tools assess fluency through timed assessments in the areas of (a) oral counting, (b) one-to-one correspondence counting, (c) number naming, and (d) quantity comparison. Though the “titles” of these subtests may vary by publisher, these measures are considered to be indicators of general number sense development.

Preschool universal screening measures are single proficiency measures, assessing indicators related to number competence. Statistically the predictive validity of these tools are not as high as have been established with similar measures used in the later grades, though correlations with other early childhood tools have determined the predictive validity to fall within a reasonable range. While there are limitations associated with these tools, the measures described below do show promise with regard to identifying young children who appear to need additional instructional support. Additionally the information gathered during the universal screening process may be used as a source of kindergarten transition information

Tools that can be used as part of the universal screening and progress monitoring process and are available to the public at this time include:

#### Preschool Numeracy Indicators (PNI)

<http://www.memphis.edu/psychology/people/faculty/rfloyd/pni/index.php> The PNI assessment has been found to be reliable and valid and sensitive to growth over time. The PNI assessment consists of four early numeracy fluency measures including oral counting fluency, one-to-one correspondence counting fluency, number naming fluency, and quantity comparison fluency. The results of the four subtests are linked to the general outcome of number sense development (Floyd, Hojnoski, & Key, 2006).



#### AIMSweb Tests of Early Numeracy

<http://www.aimsweb.com/measures-2/test-of-early-numeracy-cbm/> Information collected as a part of this assessment includes magnitude comparison, missing number, quantity discrimination, and number identification. Although the AIMSweb Tests of Early Numeracy are typically used with kindergarten and first grade, preschool norms are available through that assessment system.

#### Early Arithmetic, Reading, and Learning Indicators (EARLI)

<http://emp.sagepub.com/content/69/5/825.short> Another measure in development for preschool is the EARLI, which includes six brief measures of numeracy. This assessment has been shown to demonstrate good technical adequacy (Reid, DiPerna, Morgan, & Lei, 2009). The authors of this assessment can be contacted to ascertain availability.

Early Numeracy Measures

K-1 Early Numeracy Measures

### Universal Screening for Grades K-1

Universal screening for kindergarten and first grade also assesses skills and concepts related to number sense. Those measures include numeral recognition (Number Identification), magnitude comparison (Quantity Discrimination), and strategic counting (Missing Numbers). Of these components, strategic counting and magnitude comparison have been identified as key predictive variables (Gersten, Clarke, & Jordan, 2007). All students in kindergarten and first grade should also be screened on early numeracy skills three times a year. If using AIMSweb as a universal screener, it is also necessary to screen first and second grade students on computation skills.

The screening assessments recommended for kindergarten and first grade are: AIMSweb Tests of Early Numeracy Early Numeracy Indicators, easyCBM, and Star Math Enterprise. All of these early numeracy assessments are individually and quickly administered. For example, the AIMSweb assessments take one minute per student for each of the four subtests. Students who fail to reach the benchmark on one or more of the early numeracy subtests are grouped for instruction during the MTSS Implementation process and sorted into groups for intervention for early numeracy skills.

Oral counting and number identification are prerequisite skills for identifying missing numbers and quantity discrimination. Students who fail to reach the benchmark in strategic counting (Missing Numbers) and magnitude comparison (Quantity Discrimination) will also need to work on oral counting and number identification. However, it is possible for students to receive instruction on the concepts of strategic counting and magnitude comparison using manipulatives and representational expressions (i.e., pictures) before skills in number identification are mastered. Remember, if using AIMSweb as a universal screener, it is also necessary to screen first grade students on computation skills.

Mathematics Tasks for K-12 Universal Screening

Grades 2-12 Universal Screeners

### Universal Screening for Grades 2-12

All students in grades 2-12 should be screened 3 times per year using an MTSS-appropriate universal screener. Universal screening measures for math can be given to an entire classroom and do not require individual administration. The universal screening assessments recommended are AIMSweb, easyCBM, and STAR Math Enterprise. These assessments provide measures of skills and concepts that will be used for the instructional groupings during the MTSS implementation process.

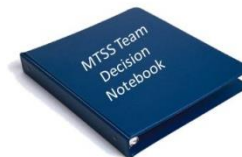
For grades 2-12, the data from the universal screening (whether it groups by Computation and Concepts/Applications or by Focal Points/Domains) will designate two groupings of students: (1) students who are at or above benchmark, and (2) students who are below

benchmark. Students who are below benchmark will be further grouped for supplemental and intensive interventions.

At this point, leadership teams need to determine possible screening assessments for each grade level. Teams will need to select the universal screening assessment that will be used and document the following universal screening decisions:

1. The screening assessment selected.
2. For which students/which grades.
3. Is this a primary source or supporting source of data?
4. Decision rules.
5. What the data are used for.
6. The areas assessed.
7. Who will administer the assessment?

This information should be documented on the Mathematics Comprehensive Assessment Planning Tool. Begin planning for data management by determining who will be responsible for entering the data and producing reports when the assessment data are collected. Consider any technical training needed for this to occur.



### **Decision Rules for the Universal Screener**

The decision rules that need to be established are the cut points/criteria on the universal screening assessments that will be used to determine which students are performing adequately as well as signal which students need supplemental or intensive support due to advanced or at-risk learning needs. For screening purposes, it is assumed that the needs of students who are performing at adequate levels will be met through differentiated instruction in the core curriculum.

Students whose scores indicate a need (advanced or at risk) will receive both differentiated instruction in the core and additional support.

### **Decision Rules for Preschool**

As discussed previously, universal screening/progress monitoring tools utilized for preschool children are similar to, but provide less information than those used in later grades. Therefore, the leadership team must determine how best to identify young children who may benefit from more instructional support, and do so in a manner that can be managed by the overall system. Criterion level benchmarks, predictive of later mathematical ability have yet to be established for this age group; therefore, information provided by these assessments primarily includes normative comparisons (national or local). As a rule of thumb, leadership teams may wish to consider providing supplemental or targeted support to young children whose scores fall at or below the 30th percentile according to national norms unless a

large number of the total classroom falls within this range. If that is the case, then the leadership team should support some type of supplemental instruction for the entire class. For programs utilizing tools that do not provide national norms, it may be necessary to use classroom norms and/or create local norms and identify an appropriate “cut-score” to meet the needs of the program. In both instances, leadership teams should establish how/if other assessment information will be used in conjunction with the universal screening data to make instructional decisions for this age group.



### **Decision Rules for Grades K-12**

The universal screening assessments are used at these grade levels to (1) identify students with advanced learning needs who may be in need of extension or acceleration opportunities and (2) identify students who require intervention. They provide specific cut points, criteria, or a range based on nationally established norms and percentiles or benchmarks. Therefore, for students at risk, the decision rules are already established to determine which students are in need of supplemental, intensive support or additional diagnostic assessment. If additional information to help select criteria is needed, the book *The ABCs of CBM: A Practical Guide to Curriculum-Based Measurement* (Hosp, Hosp, & Howell, 2007) can be a useful resource. Leadership teams will also need to determine cut-scores/criteria that will be used to identify students with advanced learning needs if the manual or technical guide for the selected assessment does not provide such. When using norm percentiles to determine intervention needs, the minimum criterion for intervention is the 25th percentile for AIMSweb and STAR Math Enterprise and the 40th percentile for easyCBM. The 10-24th percentile range for AIMSweb and STAR Math Enterprise and the 20-39th percentile range for easyCBM, includes students needing supplemental intervention and the 9th percentile and below for AIMSweb and STAR Math Enterprise and the 19th percentile and below for easyCBM, includes students needing intensive intervention. (See chart below.) Students with scores between the 25-50th percentile for AIMSweb, between the 25-39th percentile for STAR Math Enterprise, and the 40-50th percentile for easyCBM need additional instructional support. (See chart below.) This additional support can be provided through the use of differentiated instruction during core instruction or small-group intervention. This is a decision that leadership teams will need to make depending on the number of students needing intervention and the resources available.

SCREENER	ON TRACK RANGE		SUPPLEMENTAL RANGE	INTENSIVE RANGE
	No add'l help	Add'l help needed		
AIMSweb	50%ile and above	25-49%ile	10-24%ile	9%ile and below
STAR Math Enterprise	40%ile and above	25-39%ile	10-24%ile	9%ile and below
easyCBM	50%ile and above	40-49%ile	20-39%ile	19%ile and below

At this point the leadership team needs to:

1. Review any screening assessment selected to determine the cut points or decision rules provided by the test publisher.
2. Determine cut points or decision rules to be used to identify students with advanced learning needs.
3. Determine cut points or decision rules to be used to identify at-risk students in need of additional screening or diagnostic assessment.
4. Determine cut points or decision rules to be used to identify students in need of supplemental or intensive support.
5. Document all decision rules established on the Mathematics Comprehensive Assessment Planning Tool.

### Progress Monitoring Assessment

In mathematics, the assessments recommended for progress monitoring school-age children are the progress monitoring tools associated with the selected universal screener. These assessments were selected so that the progress monitoring components of the assessment (1) measure small increments of change, and (2) have sufficient multiple forms to allow for frequent administration.

Progress  
Monitoring  
Interventions:  
Math

### Progress Monitoring Assessment for Preschool

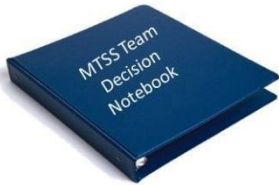
The preschool universal screening measures described previously will also be used to monitor the progress of preschool children receiving additional instructional support. At the preschool level, there is no formal guidance regarding how often to administer progress monitoring assessment probes although monthly progress monitoring is a generally accepted practice in the field. In addition to the fluency measures described, progress should also be monitored for skill mastery and may include formal/informal or teacher created measures that provide information related to error patterns and strategies utilized to solve the mathematical problem.

*Principals need to have a thorough understanding of the entire assessment process as well as why students move through it in the fashion they do.*

## Matching Progress Monitoring Assessment to Instructional Focus

Progress monitoring for students receiving supplemental and intensive instruction is critical so that teachers know how to adjust instruction and grouping. The assessment instrument chosen for progress monitoring must be able to measure the small increments of change and have multiple forms to allow for frequent administration. At this point, leadership teams need to determine the progress monitoring assessment that will be used and document the following decisions:

1. The assessment selected.
2. For which students/which grades.
3. Is this a primary source or supporting source of data?
4. Decision rules.
5. What the data is used for.
6. The areas assessed.
7. Who will administer the assessment?



This information should be documented on the Mathematics Comprehensive Assessment Planning Tool. Begin planning for data management by determining who will be responsible for entering the data and producing reports when the assessment data is collected. Consider any needed technical training for this to occur.

## Frequency of Progress Monitoring

The frequency of progress monitoring is important to consider during assessment selection to ensure that a sufficient number of multiple forms are available to allow for frequent administration. The Institute of Education Sciences (IES) recommends monitoring the progress of Tier 2, Tier 3, and borderline Tier 1 students at least once a month using grade appropriate general outcome measures. General outcome measures (such as CBMs) use a sample of items from the array of concepts covered over one year to assess student progress. They provide a broad perspective on student proficiency in mathematics. Examining student performance on these measures enables teachers to determine whether students are integrating and generalizing the concepts, skills, and strategies they are learning in the core curriculum and the intervention.

The IES also recommends the use of curriculum-based assessments (i.e., unit tests, mastery tests, or daily probes) in interventions to determine whether students are learning the skills being taught in the intervention. These measures can be used as often as every day or as infrequently as once every other week. The results of these assessments can be used to determine which concepts need to be reviewed, which need to be retaught, and which have been mastered. The IES recommends using both general outcome measures and curriculum-based assessments for students receiving interventions

Frequency of  
Progress  
Monitoring

Two Types of  
Progress  
Monitoring

(Gersten et al., 2009a). The leadership team will need to determine the frequency of progress monitoring in your building.

In addition, the leadership team needs to consider recommendations about how many data points are needed to be able to make an accurate instructional decision (a more in-depth discussion can be found in the section that follows about decision rules). Typically, three to four consecutive data points are used for making instructional decisions. With these two considerations in mind, the leadership team will need to look for progress monitoring assessments with enough alternate forms to monitor progress.

### Decision Rules for Progress Monitoring Assessments for K-12

Decision rules for progress monitoring assessments typically use several data points to determine whether the current instruction is succeeding or whether an adjustment in instruction is needed. The Four-Point Decision Making Rule is an example:

Given at least six data points, examine the last four consecutive scores to determine instructional success.

- If all four scores fall below the goal line, an adjustment in instruction is recommended.
- If all four scores fall above the goal line, a goal increase is recommended.
- If neither applies, keep collecting data until the four-point rule can be applied.  
(Hosp, Hosp, & Howell, 2007; Stecker & Fuchs, 2000)

Other researchers have recommended adjusting instruction if three data points in a row fall below the goal line or aimline (Shinn, 1989). Teams need to select a decision rule for the number of data points needed before deciding whether an adjustment in learning is needed. Decision rules also need to be established and documented for entering and exiting each tier in order to determine when an adjustment in instruction is necessary and to keep movement fluid between the tiers.

The leadership team needs to review each selected progress monitoring assessment to determine if decision rules are suggested and review other options for decision rules. Next, the team should select decision rules for frequency of progress monitoring at each tier as well as decision rules for the number of data points for data-based decision making to occur. Finally, select decision rules for:

1. Decreasing the intensity of instruction.
2. Continuing current instruction.
3. Increasing intensity of instruction.

When to Use the  
Two Types of  
Progress  
Monitoring

Decision Rules for  
Progress  
Monitoring

Ex. 4-Point  
Decision Making  
Rule





Document all decision rules established on the Mathematics Comprehensive Assessment Planning Tool.



### **Math Grouping Process for Intervention for K-12**

The math grouping process for intervention is based on research and recommendations from nationally recognized math experts. To group students for intervention, universal screening data are used to conduct the initial sorting of students into two groups: (1) at or above benchmark and (2) below benchmark. Students who score at or above benchmark receive core instruction with differentiation to meet their needs, whether it is acceleration for those well above benchmark or support for those closer to benchmark.

Students who score below benchmark on the universal screener receive core instruction with differentiation and either supplemental **or** intensive interventions. To identify the instructional focus for students in intervention, look at the results from their comprehensive protocol intervention placement test or the screener's Instructional Planning Report to determine a student's missing skills/concepts. (See the MTSS Math Grouping Process chart in the Appendix.) Placement in intervention groups is then determined using the identified instructional focus for each student and the intensity level indicated for intervention in supplemental or intensive groups. Record groups on the Instructional Grouping Worksheet for Intensive Intervention Groups and Supplemental Intervention Groups. This is addressed in Implementation Step 3.

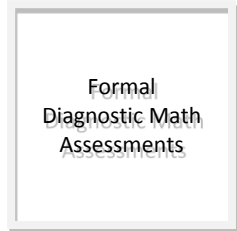
### **Math Grouping Process for Intervention for Preschool**

There is no formal process for grouping preschool children according to instructional interventions; however, the initial sorting of universal screening scores as above and below benchmark (30th percentile national norms, or as identified by the leadership team) will be used to identify children in need of additional support. The collaborative team will determine the need and type of intervention that is appropriate for individuals and groups of children according to information collected as a part of core curriculum assessments, universal screening, and diagnostic information. Depending on the information gathered and the mathematics curriculum utilized, the collaborative team may identify intervention mechanisms that best suit their program and student population. For example, if using SRA Building Blocks curriculum, the collaborative team would access the Learning Trajectories chart to identify where the child is on the learning path, set a mathematical goal on the developmental path, and differentiate instruction accordingly.

### **Formal Diagnostic Assessment for K-12**

Whenever students fail to make adequate growth in intervention, it may be an indication that further analysis is needed. The school

may decide to gather additional diagnostic information by conducting error analysis and examining error patterns (Ashlock, 2005; Riccomini, 2005), especially at the student’s instructional level. When error analysis indicates possible skill deficits, verification of these deficits can be conducted by using single-skill CBM probes (Hosp, Hosp, & Howell, 2007). Each single-skill probe assesses only one type of skill at a time, enabling a more reliable and valid assessment of specific deficits for a given computational skill. Single-skill CBM probes for mathematics are available on the MTSS website at [www.kansasmtss.org](http://www.kansasmtss.org), from Intervention Central at [www.interventioncentral.com](http://www.interventioncentral.com), or at [www.thatquiz.com](http://www.thatquiz.com) for an online version. One source for error analysis is Marilyn Burns’ Math Reasoning Inventory at <https://mathreasoninginventory.com>. The school may also choose to administer a formal diagnostic assessment to determine underlying mathematics issues. This assessment is not for special education, but rather for the purposes of planning instruction.



Formal diagnostic assessments for mathematics include:

- TEMA3 – Test of Early Mathematics Ability–Third Edition (ages 3.0 to 8.11):  
[http://portal.wpspublish.com/portal/page?\\_pageid=53,69533&\\_dad=portal&\\_schema=PORTAL](http://portal.wpspublish.com/portal/page?_pageid=53,69533&_dad=portal&_schema=PORTAL)
- Key Math III (grades K-12):  
<http://psychorp.pearsonassessments.com/HAIWEB/Cultures/en-us/Productdetail.htm?Pid=PAaKeymath3>
- Tools for Early Assessment in Math (TEAM) (grades preK-5):  
<http://www.team.mcgraw-hill.com/login/>
- Star Math Enterprise (grades 3-12): <http://www.renlearn.com/sm/>
- Number Sense Brief (grades K-3):  
<http://www.ncbi.nlm.nih.gov/pmc/articles/PMC2855153/>
- Number Knowledge Test (age levels 4, 6, 8 and 10 years):  
[http://clarku.edu/numberworlds/nw\\_TestInfo.htm](http://clarku.edu/numberworlds/nw_TestInfo.htm)

Another formal diagnostic assessment for older students is the Diagnostic Test for Pre-Algebra Math (<http://isteep.com/products/default.html>).

### Formal Diagnostic Assessment for Preschool

Formal diagnostic assessments for preschool mathematics include:

- KeyMath-Revised (KM-R):  
<http://www.pearsonassessments.com/HAIWEB/Cultures/en-us/Productdetail.htm?Pid=PAaKeymath3&Mode=summary>
- TEMA3 – Test of Early Mathematics Ability-Third Edition (ages 3.0 to 8.11)  
[http://portal.wpspublish.com/portal/page?\\_pageid=53,69533&\\_dad=portal&\\_schema=PORTAL](http://portal.wpspublish.com/portal/page?_pageid=53,69533&_dad=portal&_schema=PORTAL)

- Tools for Early Assessment in Math (TEAM) (grades preK-5):  
<http://www.team.mcgraw-hill.com/login/>
- Number Knowledge Test (age levels 4, 6, 8, and 10 years):  
[http://clarku.edu/numberworlds/nw\\_TestInfo.htm](http://clarku.edu/numberworlds/nw_TestInfo.htm)

At this point the leadership team needs to consider potential diagnostic assessments for mathematics, including:

- Evaluating reliability and validity of each diagnostic tool.
- Evaluating critical skill areas for each assessment.
- Evaluating cost of each assessment.
- Evaluating the administration time required for each assessment.
- Considering professional development requirements of each assessment.

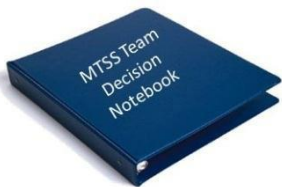
Finally, the diagnostic assessments for mathematics are selected and indicated on the Mathematics Comprehensive Assessment Planning Tool, documenting:

1. The assessments selected.
2. For which students/which grades.
3. The areas assessed.
4. Who will administer the assessment on the Mathematics Comprehensive Assessment Planning Tool.

### Decision Rules for Formal Diagnostic Assessments

Formal diagnostic assessments (norm-referenced) for mathematics provide a more in-depth analysis of a student's strengths and weaknesses and are used to further guide instruction. Most diagnostic assessments will provide either age-based or grade-based norms or rubric scoring used to determine whether or not a student has significant problems in specific skill domains. This information can then be used to design instruction specific to the student's individual learning needs. It is important that formal diagnostic assessments be given to students when additional information is needed for more customized instructional planning, but it is also important not to overuse these assessments. Formal diagnostic assessments require numerous building resources and should not be given as a matter of course to all students. Instead, they should only be given when progress monitoring data indicate further information is necessary to adequately plan instruction. Decision rules will ensure that students who need diagnostic assessment receive it in a way that is efficient and effective.

All buildings should establish decision rules to address when additional diagnostic assessments will be given, which students will receive supplemental or intensive support, and how students will be assigned to skill groups. There may be different decision rules established for the use of brief, criterion-referenced diagnostic



assessments as compared to more formal, norm-referenced diagnostic assessments that are more resource-intensive to administer. The leadership team needs to review each selected diagnostic assessment to determine skills/concepts assessed and time to administer. The team should also determine decision rules for when diagnostic assessments will be administered. In addition, the team should determine decision rules for using brief diagnostic assessments to assign students to supplemental or intensive skill groups and for moving students to another level of support. Finally, all decision rules established should be documented on the Mathematics Comprehensive Assessment Planning Tool.

### **Professional Development and Fidelity for Assessments**

Once the assessments have been selected, it is then necessary to provide appropriate professional development and ongoing support to all staff expected to use them. Decisions need to be made about who will administer, score, and interpret each assessment. If all staff members are involved in the administration of an assessment, they will need to be trained and supported in all aspects. Sometimes an assessment team might be designated to administer and score the assessments; in such cases, these team members will need to be trained and supported in all aspects. Yet it is still important for all staff members to understand what the data mean and how to interpret the instructional implications. All teachers need to be trained on the purpose, rationale, and uses of the assessment. Leadership teams may want to consider training all teachers in the assessments. Not only does this help build school capacity, but it also encourages “buy-in” of the assessment, which is critical for ensuring that teachers use the data to change their instruction. It is especially important that leadership teams provide ongoing support to teachers regarding how the assessment data are to be used instructionally.



Professional development for assessments should be integrated into the district and/or building Professional Development Plan. Professional development around each of the assessments needs to go beyond the training in the assessment to include ongoing coaching to ensure fidelity.

It is important to view the monitoring of fidelity as professional development and not punishment. The monitoring of fidelity ensures that all data are appropriately collected and used. Three main factors need to be monitored:

1. Are assessments administered and scored by staff who have been trained to do so?
2. Are assessments given according to decision rules and the assessment calendar?
3. Are assessment results correctly interpreted and used to guide instructional planning?

Training for staff is best scheduled just before the assessments are given so the scoring rules can be practiced and reinforced. Effective ways to minimize scoring errors and ensure fidelity include making sure that examiners have:

1. Excellent training.
2. Opportunities to practice.
3. Periodic ongoing training.
4. Experienced examiners to check first-time examiners' scores.
5. Opportunities to shadow score.

Within the frame of professional development, having new examiners work with experienced examiners and opportunities for shadow scoring offer the best opportunities for ongoing professional development of staff. These types of opportunities need to be included within the larger Professional Development Plan that is being implemented and monitored by the leadership team.

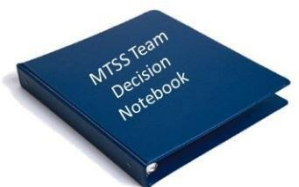
<b>TEAM DISCUSSION</b>
In planning professional development, it is helpful for the leadership team to consider these questions specific to each assessment method:
1. Which staff is expected to administer the assessment?
2. Which staff will not be administering the assessment but will be involved in interpreting instructional implications of the results?
3. Which staff members, if any, have experience with or have previously received professional development on the assessment?
4. Which staff members need to attend initial professional development on administration of the assessment?
5. Which staff members need to attend initial professional development on interpretation of the assessment?
6. When (i.e., by which date) will staff first be expected to administer the assessment?

<b>TEAM DISCUSSION</b>
7. When (i.e., by which date) will initial professional development be provided?
8. Who will provide the professional development?
9. Who will monitor the correct administration (fidelity) of the assessment?
10. What method will be used to monitor the correct administration (fidelity) of the assessment?
11. How frequently will the administration (fidelity) of the assessment be monitored?
12. Which staff member needs professional development in conducting error analysis and examining error patterns?
13. When and how will ongoing professional development for staff be provided?
14. When and how will professional development for staff needing additional support in correct administration of the assessment be provided?
15. Who will provide professional development for new staff and how will it be provided?

These questions are designed to help leadership teams as they begin the development of an overall Professional Development Plan. Once specific decisions are made, the building leadership team may record the results on the building's results-based staff development plan and/or on the Professional Development Planning Tool in the Decision Notebook. The team should consider whether the discussion of professional development and fidelity of assessment has led to a need to develop an Action Plan or to add any items to the Stop-Doing List.

### **Review Policies and Practices for Assessment**

It is important for the leadership team to review current district and building policies and practices to identify any that may require,



prevent, or otherwise influence how, when, and what assessments are given.

**Policies** are written rules or guidelines that must be followed. These guidelines can come from any level: the federal government, state government, district office, building principal, or other source. To align resources to support change, it is important to identify where the policy originated and why and if it is still functional and necessary.

*Principals will need to take an active lead in seeing that policies and practices support the assessment process.*

**Practices** are the actions that come about due to policies or tradition. Practices can be initiated because of the adoption of a new curriculum or because of staff attendance at a workshop. It is important to identify all the practices being used throughout the building even if isolated to a small number of staff because some practices may be in opposition to new practices the leadership team is trying to implement. Other practices may be well aligned and warrant sharing with the rest of the building staff.

Leadership teams will want to consider whether any policies or practices influence how assessments are currently being used. Is there a curriculum/program requirement connected to any of the current assessments? If so, a decision to change to a different assessment will need to be considered in light of existing program requirements. Discussions about policies and practices impacting assessment decisions will not be completed quickly. Teams should plan on revisiting these discussions at several leadership team meetings. Through these discussions the leadership team identifies policies and practices that are supportive and should be continued, those that are required and must be included, or those that are inconsistent with the principles and practices of the MTSS and need to be discontinued.

Now that the Comprehensive Assessment Plan has been completed, review building and district policies and practices regarding assessment. Identify whether there are policies and practices that need to be changed to align with the Comprehensive Assessment Plan. Document these on the Policies & Practices Tool. Also, consider whether discussions of practices and policies related to assessment has led to a need to develop an Action Plan or to add any items to the Stop-Doing List.

<b>TEAM DISCUSSION</b>
1. What are the policies (rules/guidelines) that require, prevent, or otherwise influence how, when, and what assessments are used?
2. What are the practices (routines/traditions) that require, prevent, or otherwise influence how, when, and what assessments are used?
3. Are there any assessment practices that might belong on the Stop-Doing List?

### **Review the Communication Plan Related to Assessments**

Communication is a key aspect of achieving buy-in and sustained change. Therefore, procedures are designed and executed to ensure regular and consistent communication about what is happening with regard to an MTSS not only among the leadership team, but also with all stakeholders. This does not have to be a large formal plan. It only needs to be as large and formal as is necessary for the leadership team to ensure that communication occurs as planned. While developing the plan, close attention is given to bidirectional communication. A one-way communication plan may get the message out, but does not allow messages to easily come back. Reciprocal communication, when appropriate, is critical if the leadership team is to achieve buy-in and support.



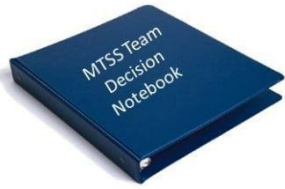
Leadership teams need to consider communication with various stakeholders regarding how, when, and what assessments are given. For example, staff will need to know of decisions regarding changes in assessment practices. Parents are interested in assessments in which their children will be participating, and leadership teams will need to discuss how assessment results will be shared with parents and possibly students. The leadership team decides what information is appropriate to share with which stakeholders and when that information should be shared.

Although the format and detail of the plan can vary, there are several questions the leadership team must consider to help ensure that the communication plan regarding assessment is carried out effectively.

- Who needs the information about assessments?
- What information do they need?
- When will communication occur?
- Who will provide the information?

*Principals need to see that the communication plan is a living document that is continuously revisited and carried out.*





- How will the communication be provided?
- Will feedback or input be requested?
- What feedback or input is needed?
- How will the feedback or input be used?

After the leadership team develops a communication plan regarding assessment, the plan should be implemented and then regularly reviewed at leadership team meetings. Any communications that have occurred or feedback that has been received can then be shared with team members, and any needed revisions can be planned and implemented. In this way, consistent communication between the leadership team and stakeholders is ensured. Document the necessary communication on the Planning for Communication Tool in the Decision Notebook.

After finalizing the Mathematics Comprehensive Planning Tool, the leadership team should review the plan for communication that needs to take place regarding assessment.



- Does the communication plan need to be modified?
- Are there steps that need to be carried out to communicate decisions about assessment?
- Consider whether the discussion of the communication plan regarding assessment has led to a need to develop an Action Plan or to add any items to the Stop-Doing List.

## Curriculum

As building teams progress through the structuring of an MTSS, curriculum is a critical topic that must be addressed. In a nation where “research exists showing that the knowledge of algebra is now essential for entry into occupations earning middle class wages” (Brown, 2009), it is imperative that schools use evidence-based curricula for a much wider range of student impact. The National Mathematics Advisory Panel (2008) delivered standards and recommendations for teaching mathematics skills/concepts that would effectively prepare students for proficiency in algebra. One important point made by the panel was that instruction in pre-kindergarten through eighth grade should be streamlined and emphasize a well-defined set of critical topics. The math curriculum should not “revisit topics year after year without bringing them to closure.” (NMAP, 2008) In other words, students must learn critical skills at a level of proficiency and fluency that enables automaticity in math computation and problem solving. Educators know that building fluency in reading is essential for reading comprehension. Likewise, proficiency and fluency in whole numbers, fractions, certain aspects of geometry, and measurement are essential for building proficiency in algebra.

The National Research Council (Kilpatrick, Swafford, & Findell, 2001) defined “mathematical proficiency” as follows:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations.
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence: ability to formulate, represent, and solve mathematical problems.
- Adaptive reasoning: a capacity for logical thought, reflection, explanation, and justification.
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

### Mathematics Core Curriculum

Because the core curriculum is the comprehensive curricula that all students receive, materials that comprise the core curriculum must support good quality classroom instruction to ensure that all students meet or exceed The Kansas College and Career Ready Standards. In order to evaluate the core curriculum materials, staff members need to analyze what materials are currently in use, examine their alignment with the Kansas College and Career-Ready Standards, look at the evidence regarding their effectiveness, and determine if there is a need to strengthen the core curriculum. Within an MTSS, this

Curriculum

What & When to  
Teach

Conceptual &  
Procedural  
Knowledge  
Interact

Core Curriculum

decision will be made by looking at the percentage of students not reaching benchmark from the universal screening data. If **40% or more** of students at a grade level are below benchmark, there is a core issue that must be addressed **before** placing students in interventions.

Research shows that the curriculum chosen for core instruction can make a difference in the achievement level attained by students. For this reason, it is important to review available evidence regarding the effects of math curricula that may be under consideration. The What Works Clearinghouse at [www.whatworks.ed.org](http://www.whatworks.ed.org) and The Best Evidence Encyclopedia at [www.bestevidence.org](http://www.bestevidence.org) are good resources for locating evidence regarding a specific math curriculum.

### **Early Numeracy in Preschool: A Foundation for Later Mathematics Ability**

A review of the research has found a direct link between the competencies supporting later mathematics ability and the development of early informal math concepts (Committee on Early Childhood Mathematics, National Research Council, 2009). Infants innately begin to apply concepts such as quantity and other implicit number concepts as they mature (Brannon, Abbott, & Lutz, 2004). As infants and toddlers grow in their ability to understand and use language, number words become part of their vocabulary. Number words can subsequently be applied in relation to objects, sets, space, and shapes. With this new and growing knowledge base, children can apply their understanding and use of number words as tools in calculation and problem solving, both of which add to a positive mathematical trajectory (Clements & Sarama, 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009). Most skills and knowledge gained by age three are acquired incidentally, with little or no formal instruction from adults (Clements & Sarama, 2007). However, some children do not enter preschool with a solid understanding of numbers. Researchers have found a high correlation between mathematical ability and the socioeconomic status of children. Children from low-income families consistently lag behind their middle-income peers in their concept of numbers (Ehrlich & Levine, 2007). As children get older, this gap widens in other areas of mathematical ability (Clements, Sarama, & Gerber, 2005).

Children from low-income families are less likely to be engaged in home environments that support mathematical learning through activities, conversations, materials, etc. In addition, children living in poverty often lag behind their peers in language ability, which has also been linked to the ease with which children learn to count. The good news is that early intervention in early numeracy skills improves the trajectory for future academic achievement (Jordan, 2010).

*Principals MUST lead by participating in the evaluation of all curricular materials.*

## Preschool Mathematics Curriculum

Although there is a clear benefit to early instruction and intervention in math, early childhood programs have historically viewed mathematics as less important than other content areas such as literacy. Less time is devoted to mathematics instruction, and what is provided is often of low quality (LaParo et al., 2008). The NCTM and the National Association for the Education of Young Children (NAEYC) affirm that high-quality, challenging, and accessible mathematics education for 3- to 6-year-old children is a vital foundation for future mathematics learning. In every early childhood setting, children should experience effective, research-based math curriculum and teaching practices. (National Association for the Education of Young Children and National Council of Teachers of Mathematics, 2002) It is important to understand that the use of play and teachable moments alone is not sufficient for the acquisition of important mathematical concepts. Organized mathematics curriculum provides opportunities for well thought out, intentional instruction and introduce mathematical concepts, methods, and language.

Core Curriculum  
for Preschool

The National Research Council recommends that preschool math experiences concentrate on (a) numbers (including whole numbers, number operations, and number relations) and (b) geometry, spatial relations, and measurement, with more math learning time devoted to numbers than to other topics. Specifically, preschool math curriculum should provide focus on:

1. Number words.
2. Counting principles, including cardinality and one-to-one correspondence.
3. Comparing, joining, and separating sets.  
(Committee on Early Childhood Mathematics, National Research Council, 2009; Jordan, 2010)

Recognizing the importance of good beginnings, the NAEYC and the NCTM released a joint position statement describing what constitutes high-quality mathematics education for children ages 3-6 and what is necessary to achieve such quality. To help achieve this goal, the position statement sets forth 10 research-based recommendations from which core curriculum and specific teaching practices should be directed for preschool-aged children (National Association for the Education of Young Children and National Council of Teachers of Mathematics, 2002):

1. Enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds.
2. Build upon children's experience and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge.

3. Base mathematics curriculum and teaching practices on knowledge of young children’s cognitive, linguistic, physical, and social-emotional development.
4. Use curriculum and teaching practices that strengthen children’s problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas.
5. Ensure that the curriculum is coherent and compatible with the known relationships and sequences of important mathematical ideas.
6. Provide for children’s deep and sustained interaction with key mathematical ideas.
7. Integrate mathematics with other activities and other activities with mathematics.
8. Provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest.
9. Actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies.
10. Support children’s learning by thoughtfully and continually assessing all children’s mathematical knowledge, skills, and strategies.

“Because young children’s experiences fundamentally shape their attitude toward mathematics, an engaging and encouraging climate for children’s early encounters with mathematics is important. It is vital for young children to develop confidence in their ability to understand and use mathematics – in other words, to see mathematics as within their reach. In addition, positive experiences with using mathematics to solve problems help children to develop dispositions such as curiosity, imagination, flexibility, inventiveness, and persistence that contribute to their future success in and out of school.” (National Association for the Education of Young Children and National Council of Teachers of Mathematics, 2002, pg. 5)

### **Mathematics Foundations for K-12**

A strong core math program should be aligned with the Kansas College and Career-Ready Standards and the critical foundations leading to proficiency in algebra. The focus for math instruction for each grade level and the critical components of a math core curriculum described below are based on recommendations from the National Mathematics Advisory Panel (2008) and the National Council for Teachers of Mathematics’ Curriculum Focal Points (National Council of Teachers of Mathematics, 2006).

### **Early Numeracy (Grades K-1)**

Students should be given explicit and direct instruction that guides them to fluency in early numeracy skills, including such skills as oral counting, number naming, strategic counting, magnitude



comparison, ordering whole numbers, joining and separating sets, describing shapes, and ordering objects by attributes.

### **Whole Numbers, Fractions, and Decimals (Grades 2-7)**

It is critical that students develop fluency with whole numbers, fractions, and decimals. In addition, students should develop a strong understanding of measurement and geometry designed to build toward fluency with algebraic concepts.

Elementary focus on:

- Critical foundations for algebra.
- Proficiency with whole numbers, fractions, geometry, and measurement.

Middle and high school focus on:

- Symbols and expressions.
- Linear equations.
- Quadratic equations.
- Algebra of polynomials.
- Finite probability.

### **Algebra (Grade 8)**

All students should be given explicit instruction and sufficient practice to become proficient with algebraic equations.

### **Advanced Math (Grades 9-12)**

Students taking advanced math courses should be provided with instruction in the fundamental concepts of function and relation, invariance, and transformation. Students should become adept at visualizing, describing, and analyzing situations in mathematical terms, and be able to prove mathematically based ideas (National Council of Teachers of Mathematics, 2000).

At this point, the leadership team needs to identify all math core curricula currently used or being considered for use. The team should identify what is being taught and at which grade levels it is taught. This can be recorded on the Taking Stock of Current Math Curriculum Tool, located in the team Decision Notebook. Determine if the core curricula are evidence based, whether they sufficiently address core knowledge and skills, whether the core knowledge and skills are included at appropriate grade levels, and whether they are taught in appropriate sequence. Identify which curricula may need to be replaced due to lack of support from the evidence base, determine the core curricula to be used, and document decisions on the grade-specific Math Comprehensive Curriculum Protocol Tool, also located in the team Decision Notebook.

*Principals should be very familiar with the NCTM Curriculum Focal Points and their appropriate transfer to the classroom.*



## K-12 Curricula for Supplemental and Intensive Intervention

When fully implementing MTSS, supplemental and intensive support is provided through a hybrid intervention model that combines a protocol and problem-solving approach to ensure a rapid response to student needs as they occur. The protocol aspect of the hybrid model requires each building to preselect a set of interventions that will be used as student data indicate a need for support beyond the core. The problem-solving aspect of the MTSS hybrid model is used to further identify and customize supports for students, especially at the intensive level.

According to the IES Practice guide “Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools,” four criteria are used to select intervention curricular materials:

1. Materials should integrate computation with solving problems and pictorial representations rather than teaching computation apart from problem solving.
2. Stress the mathematical reasoning underlying calculation methods.
3. The materials should ensure that students build algorithmic proficiency.
4. The material should include frequent review for both consolidating and understanding the links to the mathematical principles.  
(Gersten et al., 2009a, p. 20)

In addition, students who require supplemental or intensive support need a comprehensive curriculum that addresses math deficits in conceptual understanding and procedural fluency simultaneously. A good comprehensive supplemental curriculum should include:

- Curriculum-based assessments.
- A placement test.
- A systematic instruction model.

These characteristics will ensure ease in grouping students for appropriate intervention and provide a stronger intervention.

For some students, such as those needing targeted help in fractions, schools will also need to identify supplemental materials that are concept specific. Examples from the MTSS Potential Math Intervention Curricula list (see Appendix A) include:

1. Computation of Fractions—Math Intervention for Elementary and Middle Grades Students.
2. Computation of Integers—Math Intervention for Elementary and Middle Grades Students.
3. Solving Equations—An Algebra Intervention.

According to the National Mathematics Advisory Panel (2008), the ability to recall basic mathematics facts is critical for general success in mathematics. The National Mathematics Advisory Panel (2008), Common Core State Standards Initiative (2010), and National Council of Teachers of Mathematics (2006) all state that the quick and accurate recall of math facts is a core skill and prerequisite for higher level learning. Automatic recall must be developed over time through sufficient instruction, practice, and feedback (Baroody, 1999; Willingham, 2009). VanDerHeyden and Burns (2008) described this process as moving from acquisition (accuracy) to proficiency (speed). Interventions at all grade levels should devote 10 minutes in each session to building fluent retrieval of basic arithmetic facts (Gersten et al., 2009a, p. 37).

Materials targeted to match the focus of intervention instruction, whether comprehensive or concept specific, will need to be available for each group. As teams consider curricular materials, it is important to ensure that curricular materials that match the instructional needs of the students in each group are available and that the materials cover the instruction and review of basic math facts.

The first step in creating the intervention supports a building will offer is to choose curricular materials that will support supplemental and intense interventions around all essential math skills and concepts. Just as the core curriculum was reviewed and evaluated by staff, it is imperative to review current supplemental and intensive materials to determine what will work best to meet the academic needs of all students. Curricula for supplemental and intensive instruction should complement the core curricula.

Math programs for supplemental and intensive interventions need to lead to student mastery of skills/concepts. The spiral curricula found in many core math programs often lead to confusion and a lack of skill development for students who find math to be challenging (Hardy, 2005; National Mathematics Advisory Panel, 2008). This approach should **not** be used for a supplemental curriculum. Materials that provide sufficient explicit instruction and practice to lead to skill mastery should be used instead. Teams must identify intervention materials currently being used and critically evaluate them to ensure that all essential skills/concepts are represented. The materials selected will need to be identified as a comprehensive curriculum or concept specific curriculum. This review is also recorded on the Taking Stock of Current Math Curriculum Tool. By conducting this review, staff will be positioned to make necessary decisions as to whether there are gaps in materials that should be filled. Staff will also be able to make decisions about discontinuing or replacing curricula due to the lack of effectiveness or an evidence base. A list of potential math interventions that have been independently researched, show



promise, or are being used with success in Kansas are listed on the Potential Math Intervention Curriculum Tool, located in Appendix A. This is not an exhaustive list; rather, it was designed to help teams as they begin to explore intervention curricula.

### **Preschool Curricula for Intervention**

Curricula that is appropriate for meeting the more intensive needs of children falling below benchmark on the universal screening assessments must include a scope and sequence of instructional activities that is based on a progressive developmental continuum of mathematical goals, often referred to as a learning trajectory (Clements & Sarama, 2009). The area of focus for children receiving support will be object counting correspondence (one-to-one counting), oral counting, and cardinality. In many programs the same curriculum utilized in the core can be used for intervention as well. Diagnostic information can be used to compare against the learning trajectory and identify learning goals and instructional activities that are best matched to the needs of students. Such activities can be carried out in small groups, and/or in learning centers in game-like activities where students have ample time for repeated exposure to new skill instruction and substantial practice to acquire both mastery and fluency.

In summary, the leadership team needs to:

- Identify for preschool through grade 12, all supplemental and intensive curricula currently used or being considered for use and record them on the Taking Stock of Current Math Curriculum Tool.
- Determine if the supplemental and intensive curricula is evidence based and sufficiently addresses all skill and concept topics.
- Make final selections for the supplemental and intensive curricula materials to be used and document these decisions on the Math Comprehensive Curriculum Protocol Tool.
- Determine a system for organizing and using the materials selected to ensure that supplemental and intensive intervention materials are easily accessible.

### **Professional Development and Ensuring Fidelity for Curricula**

Once the curricular materials have been selected, it is then necessary to provide appropriate professional development and ongoing support to all staff members expected to use the curricula to provide instruction. Simply having curriculum materials available at each level (i.e., core content, supplemental, intense) does not ensure their appropriate use.

Curricula for supplemental and intensive intervention should be implemented as described in the manual. For example, some math intervention programs require 45 minutes of instruction five days a



week for fidelity in order to achieve desired results. Not following the intervention as described is highly likely to reduce its overall effectiveness. Staff must have a working knowledge of the curriculum content and materials as well as an understanding of the planning and pacing process for lesson development. The building leadership team must set clear expectations that curricular materials will be implemented and used with fidelity and provide professional development to support this occurring.

Professional development activities must be differentiated in order to support the individual needs of staff members as they acquire the necessary knowledge and skills enabling them to implement the specified curriculum with fidelity. Initial and ongoing training should be differentiated based upon expectation of use, alignment of materials, and prior knowledge of the content area and should build on prior professional development activities.

The Professional Development Plan for curricula implementation is dynamic in nature and results in the curriculum being implemented with fidelity. It is a plan that proactively identifies activities that are based on individual staff learning needs and will result in the knowledge and skills necessary to utilize the curriculum. It ensures that staff are accessing and utilizing curricular materials in the expected manner by planning for and conducting intermediate and follow-up activities. To accomplish this, the building leadership team establishes methods for monitoring the use of the curriculum by individual teachers. The information collected is utilized to differentiate ongoing professional development and support for each staff member.

*The principal must be actively involved in establishing a fidelity-ensuring process and using the produced data within the self-correcting feedback loop to drive professional development and instructional decisions.*

Activities for monitoring the fidelity of curriculum implementation by individuals are not intended to be punitive, but rather should be understood as a piece of the overall Professional Development Plan, resulting in further staff support as needed in order to positively impact student outcomes. To accomplish this, a method to check for correct use of the curricular materials needs to be established. Many purchased curricula and programs come with fidelity monitoring tools such as observation or walk-through forms. It is the building leadership team's responsibility to establish a plan to monitor and support the correct and effective use of curricular materials.

### **Planning Professional Development**

The building leadership team will identify the professional development needs related to curriculum implementation by identifying the staff members who will be teaching each curriculum, considering the characteristics of each specified curriculum, reviewing previously provided professional development, and identifying staff members who will need additional support to implement the curriculum with fidelity.

**Core Curriculum:** It is important that all staff with instructional responsibility have a solid understanding of the core curriculum and receive professional development that enables them to implement it with fidelity. All staff, in this instance, means staff responsible for instruction at all three levels of the MTSS. This is necessary to ensure that the curricula implemented at the supplemental and intensive level are aligned to the core curriculum.

**Supplemental and Intensive Curricula:** It is not necessary that all staff in a building know how to implement the supplemental and/or intensive curricula; however, it is important that collaborative team members understand the skills/concepts targeted in each curriculum so they can be involved in appropriate instructional planning.

In planning professional development it is helpful for the leadership team to consider these questions specific to each curriculum selected:

<b>TEAM DISCUSSION</b>
1. Which staff members are expected to implement the curriculum?
2. Which will not be implementing the curriculum but will be expected to align instruction with it?
3. Which staff members, if any, have experience with or have previously received professional development on the curriculum?
4. Which staff members need to attend initial professional development on the curriculum?
5. Who will provide the professional development?
6. Who will ensure that staff members have all materials necessary to implement the curriculum, and how will they know?
7. What method will be used to monitor the use/implementation (fidelity) of the curriculum?
8. How frequently will the use/implementation (fidelity) of the curriculum be monitored?

<b>TEAM DISCUSSION</b>
9. When and how will ongoing professional development for staff using the curriculum be provided?
10. When and how will professional development for staff needing additional support to use the curriculum effectively be provided?
11. Who will provide professional development for new staff, and how will they provide it?

These questions are designed to help leadership teams as they begin the development of an overall Professional Development Plan. Once specific decisions are made, the building leadership team may record the results on the building’s results-based staff development plan and/or on the Professional Development Planning Tool.

At this point the leadership team needs to review the choices listed on the Math Comprehensive Curriculum Protocol Tool and identify the individuals who will be responsible for providing each curriculum, professional development, coaching, etc.

Consider whether your discussion of professional development and fidelity of assessment has led to the need to develop an Action Plan or to add any items to the Stop-Doing List.

**Review Policies and Practices for Curriculum**

Now that the Math Comprehensive Curriculum Protocol Tool has been completed, review district and building policies and practices regarding curriculum. Identify whether there are policies and practices that need to be changed to align with the Tool. Document these on the Policies & Practices Tool.



Consider whether the discussion of policies and practices regarding curriculum has led to a need to develop an Action Plan or to add any items to the Stop-Doing List.

<b>TEAM DISCUSSION</b>
1. Are there any policies (rules/guidelines) that require, prevent, or otherwise influence how, when, and what curriculum are used for core instruction?
2. What are the practices (routines/traditions) that require, prevent, or otherwise influence how, when, and what core curriculum is used?
3. Are there any policies (rules/guidelines) that require, prevent, or otherwise influence how, when, and what curricula are used for intervention?
4. What are the practices (routines/traditions) that require, prevent, or otherwise influence how, when, and what curricula are used for intervention?
5. Are there any practices that might belong on the Stop-Doing List?

Review  
Communication  
Plan

Monitoring  
Fidelity of Paper  
Implementation

### **Review Communication Plan as Related to Curriculum**

Review the plan for communication that needs to take place regarding curriculum.

- Does the communication plan need to be modified?
- Are there steps that need to be carried out to communicate decisions the leadership team has made about curricula, especially to school staff?
- Consider whether the discussion of a communication plan for curriculum has led to the need to develop an Action Plan or to add any items to the Stop-Doing List.

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## Instruction

Teachers need strategies for organizing both instruction and study of material to facilitate students learning and remembering information, and to enable students to use what they have learned in new situations. The Institute of Education Sciences offers seven recommendations that “reflect ... some of the most important concrete and applicable principles to emerge from research on learning and memory (Pashler et al., 2007). (The IES Practice Guide can be accessed at: <http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=1>.)

The seven recommendations for organizing instruction and study from the practice guide are:

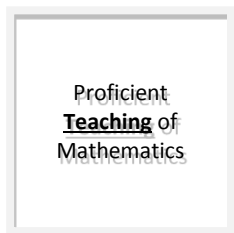
1. Space learning over time.
2. Interleave worked example solutions with problem-solving exercises.
3. Combine graphics with verbal descriptions.
4. Connect and integrate abstract and concrete representations of concepts.
5. Use quizzing to promote learning.
6. Help students allocate study time efficiently.
7. Ask deep explanatory questions.

### Instruction of Mathematics

Instruction of mathematics is characterized by comprehension of ideas; ready access to skills and procedures; an ability to formulate and solve problems; a capacity to reflect on, evaluate, and adapt one’s knowledge; the ability to reason from what is known to what is wanted; and a habitual inclination to make sense of and value what is being learned. Just as mathematical proficiency involves interwoven strands, teaching for mathematical proficiency requires similarly interrelated components. In the context of teaching, proficiency requires:

- Conceptual understanding of the core knowledge required in the practice of teaching.
- Fluency in carrying out basic instructional routines.
- Strategic competence in planning effective instruction and solving problems that arise during instruction.
- Adaptive reasoning in justifying and explaining one’s instructional practices and in reflecting on those practices so as to improve them.
- A productive disposition toward mathematics, teaching, learning, and the improvement of practice.

Like the strands of mathematical proficiency, these components of mathematical teaching proficiency are interrelated. It is not sufficient



that teachers possess only core knowledge. One of the defining features of conceptual understanding is that knowledge must be connected so that it can be used intelligently. Teachers need to make connections within and among their knowledge of mathematics, students, and pedagogy.

*Principals need to understand these research-based instructional strategies and be able to recognize them in practice within all instruction.*

The kinds of knowledge that make a difference in teaching practice and in students' learning are an elaborated, integrated knowledge of mathematics, a knowledge of how students' mathematical understanding develops, and a repertoire of pedagogical practices that take into account the mathematics being taught and how students learn it (National Research Council, 2001, p. 380).

Teachers at all grade levels (preschool through grade 12) must be prepared to provide strong initial instruction in critical skills and conceptual knowledge to their classroom as a whole and to small groups for differentiated instruction and intervention instruction. In every grade, specific skills must be taught and mastered. **Teachers must be able to support student growth in critical areas by providing research-based instructional strategies** that include explicit and systematic instruction, ample practice opportunities, scaffolding techniques and differentiated instruction to meet students' instructional needs. This also includes providing models of proficient problem solving along with opportunities for students' verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Evidence-Based  
Instructional  
Practices

### Explicit Instruction

In preschool programs, explicit instruction refers to intentionally designed learning situations that facilitate acquisition of very specific mathematical skills or concepts. Explicit instruction can be implemented and embedded within a wide variety of preschool activities including whole group, small group, learning centers, and free play as well as general classroom routines such as arrival/departure, snack, and transition times (Committee on Early Childhood Mathematics, National Research Council, 2009).

Explicit  
Instruction

Explicit instruction for school-age children means that students are told what they will learn and are given the procedural knowledge to learn. In practice, explicit instruction means that the teacher gives three types of instruction:

- Declarative: The teacher tells the students what concept or strategy they need to learn.
- Procedural: The teacher explains and models how to use the concept or strategy.
- Conditional: The teacher explains when the student will use the concept or strategy.

(Ellis, Worthington, & Larkin, 1994; Pearson & Dole, 1987)



Explicit instruction for students who struggle in math is effective in improving student learning.

The National Mathematics Advisory Panel (2008) recommends that instruction during interventions should be explicit and systematic. Instruction during intervention should:

- “Ensure that instructional materials are systematic and explicit. In particular, they should include numerous clear models of easy and difficult problems, with accompanying teacher think-alouds.” (Gersten et al., 2009a, p. 22)
- “Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.” (Gersten et al., 2009a, p. 23)
- Ensure that instructional materials include cumulative review in each session. (Gersten et al., 2009a, p. 24)

The National Mathematics Advisory Panel goes on to recommend that struggling students receive some explicit instruction regularly and that some of the explicit instruction ensure that students possess the foundational skills and conceptual knowledge necessary to understand their grade-level mathematics. Districts should pay close attention when selecting materials to ensure they reflect this recommendation.

### **Systematic Instruction**

Systematic instruction means that teachers provide instruction in a step-by-step manner with careful planning of the instructional sequence, including the sequence of examples. This increases the likelihood of early success with new concepts and problems that can then be supported by sequencing examples of increasing complexity. This ensures that students have an opportunity to apply their knowledge to a wide range of material and promotes the transfer of knowledge, or generalization, to unfamiliar examples (Jayanthi, Gersten, & Baker, 2008). The National Mathematics Advisory Panel suggests that teachers verbalize the solution to problems as they model problem solving for students. Tournaki (2003) assessed this approach by comparing a group of students whose teacher had demonstrated and verbalized an addition strategy against a group of students whose teacher did not verbalize a strategy; the effects were significant, positive, and substantial in favor of the students whose teacher had verbalized a strategy. (Note that, during the intervention, students with the teacher verbalizing a strategy were also encouraged to verbalize the problem-solving steps, which may also have been a factor in the success.)

Systematic instruction should include ample practice opportunities that are planned for reinforcement of previously taught skills/concepts. Practice should:

Systematic Instruction

*Principals also need to ensure these research based instructional strategies are reflected in curricular materials.*

- Provide opportunities to apply what students have been taught in order to accomplish specific tasks.
- Follow in a logical relationship with what has just been taught in the program.
- Provide students with opportunities to independently apply previously learned information once skills are internalized.

(Moats, 2005)

### **Scaffolded Instruction**

Scaffolded instruction is "the systematic sequencing of prompted content, materials, tasks, and teacher and peer support to optimize learning" (Dickson, Chard, & Simmons, 1993). When students are learning new or difficult tasks, they are given more assistance. As they begin to demonstrate task mastery, the assistance or support is decreased gradually in order to shift the responsibility for learning from the teacher to the students. Thus, as the students assume more responsibility for their learning, the teacher provides less support.



For students to become proficient in performing mathematical processes, explicit instruction should include scaffolded practice, where the teacher plays an active role and gradually transfers the work to the students. This phase of explicit instruction begins with the teacher and students solving problems together. As this phase of instruction continues, students should gradually complete more steps of the problem with decreasing guidance from the teacher. Students should proceed to independent practice when they can solve the problem with little or no support from the teacher. (Gersten et al., 2009a, p.23).

### **Opportunities to Think-Aloud**

One aspect of instructional practice extensively supported by the research is the provision of ample opportunities for practice. The National Math Advisory Panel (2008) recommends that teachers provide students with opportunities for extensive practice with feedback. Students need supported opportunities to apply what they have been taught in order to accomplish specific math tasks. They need multiple opportunities to independently apply previously learned information once skills/concepts are mastered. Students should be asked to verbalize the strategies used to complete the steps of the process along with their reasoning. Sharing how students think through solving problems with the teacher, interventionist, and the rest of the group can facilitate the development of a common language to be used when talking about mathematical problem solving.



The National Mathematics Advisory Panel suggests that districts select instructional materials that also provide interventionists with sample think-alouds or possible scenarios for explaining concepts and working through operations. Chosen intervention curricula should

help interventionists model or think through both difficult and easy examples.

### **Specific and Extensive Feedback**

“Teachers should give specific feedback that clarifies what students did correctly and what they need to improve” (Tournaki, 2003). Students should be given opportunities to correct their errors with the teacher or interventionist, asking simple guiding questions and prompting the student to discover and correct their errors. The teacher or interventionist may also need to periodically reteach or clarify prior instructions when students are not able to recognize and correct their errors. Limitations in working memory can often hinder math performance. Practice can offset this by improving automatic recall. Learning is most effective when practice is combined with instruction on related concepts because conceptual understanding promotes the transfer of learning to new problems and results in better long-term retention (Riccomini, 2010).

### **Materials Include Cumulative Review in Each Section**

In addition to practicing recently learned material, it is important that teachers provide frequent cumulative review (Gersten et al., 2009a) to promote long-term retention of concepts and skills. Cumulative reviews provide students with opportunities to practice topics previously learned in greater depth. According to the National Mathematics Advisory Panel (2008), this type of review can ensure that the knowledge is maintained over time and helps students see connections between various mathematical ideas.

The authors of the IES Practice Guide: Organizing Instruction and Study to Improve Student Learning (<http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=1>) state that teachers should “take every opportunity to prompt students to retrieve information.” (Pashler et al., 2007) It is recommended in the practice guide “that teachers use ‘closed-book’ quizzes or tests as one method for re-exposing students to key course content ... (as) a delayed re-exposure to course content helps students remember key information longer.” (Pashler et al., 2007)

### **Differentiated Instruction**

Differentiated instruction is an organized way of proactively adjusting teaching and learning to meet kids where they are and help them to achieve maximum growth as learners. Differentiation of teacher-directed instruction is a teacher’s response to learners’ needs guided by general principles of differentiation, such as the use of data, sequence of instruction, flexible grouping, materials and resources, and teachers and coaches collaborating in planning. It involves using multiple approaches to content, process, product, and learning environment (Tomlinson, 1999). Teachers can differentiate instruction



by content (what students learn), process (how students learn), product (how students demonstrate what they learn), and learning environment (the “climate” of the classroom) (Tomlinson, 1999).

### Differentiating Content

- Presenting information at various levels of difficulty.
- Presenting ideas through both auditory and visual means using peer tutors.
- Meeting with small groups—re-teaching or extending content.

### Differentiating Process

- Tiered activities: All learners work with same important information and skills/concepts, but proceed with different levels of support, challenge, or complexities.
- Provide interest centers that encourage students to explore subsets of class topic.
- Develop personal agendas.
- Provide manipulatives or other hands-on supports.
- Vary length of time a student may take to complete a task.

### Differentiating Product

- Provide options of how to express information learned.
- Use rubrics that match and extend varied skill levels.
- Allow students to work alone or in small groups for products.
- Encourage students to create their own product assignments.

### Differentiating Environment

- Provide places to work around the room that are quiet or invite collaboration.
- Provide materials that are culturally sensitive.
- Set clear guidelines for independent work that matches student needs.
- Develop routines that allow students to get help when the teacher is not available (working in small groups).
- Help students understand that some learners need to move around while others sit quietly.

When differentiating instruction, first determine the students’ readiness based on formative assessments, then determine the students’ interests and use this information to design instruction and monitor student progress (Tomlinson & Allan, 2000).

The following are some examples of differentiation practices for K-12 math:

### Tiered assignments

“In a unit on measurement, some students are taught basic measurement skills, including using a ruler to measure the length of

objects. Other students can apply measurement skills to problems involving perimeter.” (Access Center, n.d.)

### Interest centers

“Centers can focus on specific math skills/concepts, such as addition, and provide activities that are high interest, such as counting jelly beans or adding the number of eyes on two aliens.” (Access Center, n.d.)

As the leadership team begins to consider important elements for the instruction of mathematics, it is important to remember that decisions about interventions and instructional practices need to be based on research results and other high-quality evidence. The tiered levels of instruction must be systematic and carefully designed to provide the instruction that all students need.

### **Differentiated Instruction: A Different Approach**

To successfully differentiate instruction in a core classroom, teachers need manageable strategies that meet the needs of most of their students at the same time. Marian Small in her book, *Good Questions: Great Ways to Differentiate Secondary Mathematics Instruction*, advocates using two core strategies to effectively differentiate instruction to suit all students:

1. Open questions.
2. Parallel tasks.

### Open Questions

Meeting the needs of the varied students in a classroom during instruction becomes manageable if the teacher can create a single question or task that is inclusive in allowing not only for different students to approach it by using different processes or strategies, but also for students at different stages of mathematical development to benefit and grow from attention to the task.

A question is open when it is framed in such a way that a variety of responses or approaches are possible. The same question is given to the entire class; however, the question is designed to allow for differentiation of response based on each student’s understanding. All students can participate fully and gain from the discussion in the classroom learning community. In this way, all students participate fully in the class.

Strategies for creating open questions:

- Turn around a question.
  - From: What is half of 20?
  - To: 10 is a fraction of a number, what could the fraction and number be?

Differentiated  
Instruction-A  
Different  
Approach

Open Questions

- Ask for similarities and differences.
- Replace a number with a blank.
- Ask for a number sentence.
  - Q: Create a sentence that includes the numbers 3 and 4 along with the words “and” and “more.”
  - A: 3 and 4 together are more than 6.
- Change the question.
  - From: What number has 3 hundreds, 2 tens, 2 thousands, and 4 ones?
  - To: You can model a number with 11 base ten blocks, what could the number be?

### Parallel Tasks

Parallel tasks are sets of two or more related tasks or activities that explore the same big idea but are designed to suit the needs of students at different developmental levels while still getting at the same big idea and being close enough in context that they can be discussed simultaneously.

To create parallel tasks, think about how students in the class differ developmentally and develop similar enough contexts for the various options that common questions can be asked.

Example Parallel Tasks:

Option 1: An object has a length of 30 cm. What might it be?

Option 2: An object has an area of 30 cm<sup>2</sup>. What might it be?

Option 1: Create a word problem that could be solved by multiplying two one-digit numbers.

Option 2: Create a word problem that could be solved by multiplying two numbers between 10 and 100.

(Adapted from *Good Questions: Great Ways to Differentiate Mathematics Instruction* by Marian Small, NCTM, 2009)

### **Differentiated Instruction – Preschool**

A wide range of knowledge/skills may be considered to be age appropriate for a young child at any point in time. This range can be accounted for in part by differences in biological maturation, home experiences, cultural backgrounds, individual interests, and other environmental factors. In addition, some preschool programs serve a mixed-age population (e.g., 3-5 years), increasing the probability of a wide range of skills and abilities. To meet the needs of young children, including those who may require more explicit, focused, and intensive instruction, preschool programs should utilize differentiated instruction through small, flexible groups and embedded instruction within learning centers or other activities.



### Small Flexible Groups

Using information collected during the universal screening process and other assessment data (e.g., curriculum-based assessment, teacher observations, checklists), children with similar mathematical skills can be sorted into small, flexible groups of two to five children. The number of children in the group may vary depending on the individual needs of children, and groups may consist of children with skills below instructional target, at instructional target, and above instructional target. Using the core curriculum as a starting point, small-group lesson plans should reflect adaptation through the use of different materials, altering of activity, and increased or decreased levels of scaffolding.

### Learning Centers and Embedded Learning Opportunities

To increase opportunities for practice and/or provide opportunities for even more individualized support, learning centers can be proactively designed based on the individual learning needs of the children. Opportunities for extended practice or individualized instruction can be embedded within typical learning centers (e.g., blocks, pretend play/drama, and art) or may be specially designed for instruction of a specific math concept. These embedded learning opportunities should be designed to complement and extend what was learned in the small groups.

## **Core Instruction**

### Grades K-12

The core curriculum for mathematics should be based on the Kansas College and Career-Ready Standards and include the following instructional practices:

- Plan for learning goals that are aligned to the Kansas College and Career-Ready Standards.
- Manage classroom discourse using questioning and discussion strategies.
- Link experience and background knowledge to the abstract through written and oral communication.
- Provide for instruction in conceptual understanding, computational and procedural fluency, and problem solving, as these skills are equally important and mutually reinforce each other.
- Provide for heightened interactions and considerations of various problem-solving models with varied student groupings.
- Apply the strategic use of manipulatives, calculators, and computers to develop students' mathematical fluency.
- Use formative assessments to plan for adjustments and modifications to the learning plan.



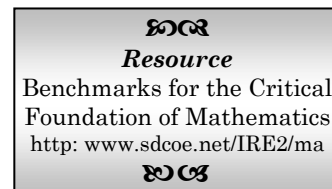
*Principals need to ensure that core curriculum is aligned vertically and horizontally.*

- Provide differentiated instruction to respond to student needs (Kilpatrick, Swafford, & Findell, 2001; National Mathematics Advisory Panel, 2008).

Mathematics core instruction should occur for a minimum of 50 to 60 minutes per day (National Council of Teachers of Mathematics, 2006; Riccomini & Witzel, 2010). The math core should be effective with the majority of students and include differentiated instruction for students who display difficulty with math, yet may meet the benchmark standard. The NCTM (2006) recommends that students be enrolled in a mathematics course each year in grades Kindergarten–12. Topics of study in kindergarten through eighth grade should be well established and streamlined to build proficiency in the early grades. Students should exhibit proficiency in using whole numbers, fractions, and decimals by the end of sixth grade and be prepared for and offered algebra instruction by eighth grade (National Mathematics Advisory Panel, 2008). A chart of the Benchmarks for the Critical Foundation of Mathematics can be found in the National Math Advisory Panel Final Report (2008).

One of the ongoing issues within the instruction of mathematics has been whether instruction should be student centered or teacher directed. Research does not support the use of either instructional approach alone. Rather, different forms of instructional practices can have a positive impact under certain circumstances (National Mathematics Advisory Panel, 2008). Students who struggle in mathematics need explicit and systematic instruction within the core as well as during math intervention time. Such practices should include clear examples and models, detailed feedback, and the use of think alouds (Gersten et al., 2009a; National Mathematics Advisory Panel, 2008). Teacher think alouds are a way of verbalizing steps so that students can follow the entire problem-solving procedure and begin to build an automatic problem-solving repertoire. Teachers should also “periodically encourage students to ‘think aloud’ in speaking or writing their explanations as they study the material.” (Pashler et al., 2007) In this way, think alouds also serve as a basic form of error analysis for teachers to gain information as to why the student is not mastering a specific skill or concept.

It is important that students practice skills and concepts correctly. It is recommended in the IES Practice Guide: Organizing Instruction and Study for Student Learning (<http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=1>) that “instead of giving students a list of problems to solve as a homework assignment, teachers should provide a worked out solution for every other problem on the list.” (Pashler et al., 2007) The recommendation is “that teachers interleave worked example solutions and problem-solving exercises – literally alternating between worked examples demonstrating one possible solution path and problems that the





student is asked to solve for himself or herself – because research has shown that this interleaving markedly enhances student learning.” (Pashler et al., 2007) However, “as students develop greater expertise, decreased example use and correspondingly increased problem solving appears to improve learning. Gradually ‘fading’ examples into problems, by giving early steps in a problem and requiring students to provide more and more of the later steps as they acquire more expertise with the problem type, also seems to benefit student learning.” (Pashler et al., 2007)

### **Chunking**

Another instructional practice that is important when planning math instruction is the use of chunking. Teachers can use chunking to break the information being presented into smaller, more manageable sets of skill-related or concept-related groupings. This instructional practice is especially helpful for students with limited working memory and should be applied to planning both initial instruction and review and practice (Gersten et al., 2009a).

### **Peer Tutoring**

Along with scaffolding and differentiated instruction, one of the core instructional practices that are especially helpful for students who struggle with math is peer tutoring (Hall & Stegila, 2003). Many variations of peer tutoring exist, ranging from “pair and share” to more formal cooperative learning structures. When working with students having difficulty, most studies implement groups of two students working together to solve problems. Cross-age tutoring shows somewhat more promise than same-age tutors; however, it poses more logistical difficulties. Peer-assisted learning is effective for students with differing mathematical abilities as it provides reinforcement for students with higher levels of proficiency and provides scaffolding and verbal interaction for students having difficulties (Gersten et al., 2009a). The most important ingredient for success in using peer-assisted learning is that teachers must teach parameters and processes as well as the role each student plays in the process. Effective peer tutoring uses a very structured arrangement and is not simply putting students together to work on math.

Reciprocal peer tutoring (RPT-M), another term used to further define peer-assisted learning, has been shown to be effective. Although it uses the same format, it is designed to help students work effectively in a group context (Fantuzo, King, & Heller, 1992). Studies have been conducted within 30-minute time frames, including 20 minutes for peer-tutoring sessions and 10 minutes for individual practice and checking. RPT-M improved math computation skills and also the social competencies of classroom behavior, students’ perceptions of their academic abilities, peer acceptance, and academic motivation. These social competencies were also improved in the Peer Assisted

Learning Strategies (PALS) strategy. PALS is an example of a highly structured reciprocal peer-tutoring model designed for use in the regular classroom to achieve supplemental intervention (Fuchs, Fuchs, & Karns, 2001). PALS uses the following protocol:

1. Mediated verbal rehearsal when the tutor models and gradually fades the procedural steps to solving a problem.
2. Step-by-step feedback by the tutor.
3. Verbal and written interaction between the tutor and pupil.
4. Application of strategies by the pupil following tutoring session.
5. Reciprocity as student takes on both roles.

### **Supplemental and Intense Instruction for Mathematics for K-12**

Supplemental and intense instruction is designed to meet the needs of students by providing interventions. For advanced learners, intervention may include instructional support for enrichment or extension of skills/concepts already learned through instruction in the core. Struggling students receive supplemental or intense support when the universal screening data indicate that the students are not meeting their grade-level benchmark. These students will immediately be placed in a comprehensive protocol intervention. Students not demonstrating appropriate growth through progress monitoring of the comprehensive protocol intervention will then receive a formal diagnostic assessment to determine the appropriate intervention and intensity. This process is taught during MTSS Implementation training.

Supplemental support is delivered through small-group instruction with flexible groups of 6-8 students working on the same skill focus (Gersten et al., 2009b). Although supplemental instruction needs to complement core instruction, it is critical that supplemental supports not be a homework session or study hall where students are merely tutored on what was learned in core instruction. It must be separate, additional, targeted, teacher-led, explicit instruction focused on the skill deficits identified with data (Riccomini & Witzel, 2010). Intensive support may be provided through either small group or individualized instruction, but it is provided in groups with a smaller teacher–student ratio than supplemental instruction, with small groups of 1-5 students.

Many research-based and evidence-based instructional practices are effective for supplemental and intensive instruction, including:

- Explicit instruction for word problems (Montague, Warger, & Morgan, 2000; Wilson & Sindelar, 1991).
- Explicit instruction for computation (Tournaki, 2003).
- Visual representations for word problems (Owen & Fuchs, 2002).

Supplemental  
and Intensive  
Instruction

How Intensive  
Intervention  
Differs from  
Supplemental

Core,  
Supplemental,  
and Intensive  
Instruction

*Principals must develop and implement schedules that support and protect the research findings related to appropriate group size.*

- Meta-cognitive strategy instruction for fraction computation (Hutchinson, 1993).
- Schema-based instruction for problem solving (Xin, Jitendra, & Deatline-Buchman, 2005).
- Concrete-representational-abstract (CRA) sequence of instruction for:
  - Fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003).
  - Algebra (Witzel, 2005).
  - Computation (Miller & Mercer, 1993).

Both the National Mathematics Advisory Panel (2008) and the RtI Math Practice Guide Panel (Gersten et al., 2009a) also support these instructional practices. More information about these research-based instructional practices is provided below.

### **Concrete/Representational/Abstract Instruction (CRA)**

A major problem for students who struggle with mathematics is weak understanding of the relationships between the abstract symbols of mathematics and the various visual representations. Student understanding of these relationships can be strengthened through the use of visual representations of mathematical concepts. Such representations may include number lines, graphs, simple drawings of concrete objects such as blocks or cups, or simplified drawings such as ovals to represent birds. A major goal of interventions should be to systematically teach students how to develop visual representations and how to transition these representations to standard symbolic representations used in problem solving. (Gersten et al., 2009a, p.30) Such connections are the basis for conceptual understanding.

Visual representations need to be relevant to the processes or concepts that are being taught, and it is important to make connections between steps in the symbolic procedures and how they are represented in visual representations. A technique called “concreteness fading” can be used to capitalize on the benefits of CRA. In this technique, “concrete representation is used to introduce a concept or principle, and then relevant components of the concrete representation are systematically replaced with abstract representations to help learners with a range of abilities and prior knowledge to master abstract representation of the concept.” (Pashler et al., 2007)

In lower elementary grades, use concrete objects more extensively in the initial stages of learning to reinforce the understanding of basic concepts and operations. In the upper grades, use concrete objects when visual representations do not seem sufficient in helping students understand mathematics at the more abstract level. Use manipulative expeditiously, and focus on fading them away systematically to reach the abstract level. (Gersten et al., 2009a, p. 32)

Regardless of age, students benefit from instruction that moves from concrete (drawing pictures or using manipulatives) to representational (using tally marks or number lines) to abstract (using mathematical symbols). This process allows students to make connections that promote a deeper understanding of key concepts. The use of visual representations (e.g., the CRA sequence) helps students who have difficulty with computational operations or facts (Maccini & Gagnon, 2000; Miller & Mercer, 1993).

CRA instruction is suited for the following concepts:

• Early number relations	• Measurement	• Fractions
• Place value	• Geometry	• Decimals
• Computation	• Probability	• Percentage
• Number bases	• Statistics	• Money

### Schema-Based Instruction (SBI)

As recommended in the IES Practice Guide, students who have difficulties in mathematics typically experience severe difficulties in solving word problem related to the mathematics concepts and operations they are learning. This is a major impediment for future success in any math-related discipline. The Guide recommends that interventions include systematic explicit instruction on solving word problems using the problems' underlying structure. Research demonstrates that instruction on solving word problems based on underlying problem structure leads to statistically significant positive effects on measures of word problem solving. (Gersten et al., 2009a, p.26)

SBI is a research-based strategy for math problem solving in which students learn to recognize the schema or type of problem and associate it with a previously learned problem-solving framework. A schema serves the function of knowledge organization and can link numerical relationships to numerical operations. SBI typically includes instruction in both conceptual and procedural knowledge. Diagrams are used to represent information in word problems to help students figure out what operation is needed to solve the problem (Fuchs & Fuchs, 2002; Jitendra, 2002; Jitendra, 2007; Seethaler, Powell, & Fuchs, n.d.).

### Learning Strategies

Learning strategies are divided into two categories:

1. Cognitive strategies, including graphic organizers, checklists, note-taking, picture drawing, and asking questions.
2. Meta-cognitive strategies, which include self-regulating, goal setting, self-questioning, and self-monitoring.

Teaching learning strategies is critical for:

- Helping students learn and remember key concepts and become more engaged.
- Helping students become more independent by teaching them how to learn.
- Giving students greater confidence in their abilities.

Although providing students with a number of strategies can help them, students can also be overwhelmed with the introduction of too many strategies in too short of a timeframe. Strategies should be introduced systematically and aligned with the type of math problem to be solved and should also align with identified needs of the students. Specific examples of various learning strategies are discussed below.

### **Cognitive Strategy Instruction (CSI)**

According to Montague (1997), CSI provides: (1) scaffolding with systematic modeling of active thinking, (2) interactive dialogue among peers, (3) schedule of practice with transfer tasks, and (4) routines for students to verbalize their rationale for selecting a particular strategy. The teacher explicitly teaches strategies with a gradual release to the student for applying the strategy. Multiple strategies must be systematically taught, with time provided for practice and application for each strategy prior to exposing students to situations that require self-selection of a menu of problem-solving strategies. Examples of CSI include:

- Multiplication Attack Strategy, which combines representations (marks) with skip counting to teach multiplication facts; it also provides the necessary prerequisite skills.
- Subtraction Strategies, in which various strategies are provided, including count back, add to check, show with objects, use a picture, doubles (add and subtract), counting up, and zero fingers.
- Cover, Copy and Compare, in which students are taught a five-step procedure to increase math fact fluency and self-evaluate their responses.
- Rules to Lower the Amount of Memorization in Math, where rules are provided for learning addition, subtraction, multiplication, and division facts.

### **Mnemonics**

Mnemonics assist students who have difficulty in memorizing the steps to a mathematical procedure or strategy by using the initial letters of a multi-step process. They do not replace conceptual understanding. Examples of the following strategies can be found in Resource Math Strategies and in Resource Math Instruction, both available from your Recognized MTSS Trainer.

- Draw and Fast Draw: The Draw strategy was developed to help students commit multiplication facts to memory; Fast Draw helps students make the transition from visual representations to abstract numbers.
- Slobs & Lamps: These strategies assist students in remembering how to regroup. “Slobs” applies to subtraction and “Lamps” applies to addition.
- STAR: This is a number-writing strategy that helps students recall, recite, and write numerals. The information in Resource Math Strategies provides the sequence for teaching STAR to students.
- 4Bs: This mnemonic helps students remember four steps in subtraction and stands for begin (in the ones column), bigger (which number is bigger), borrow (if the bottom number is bigger, then borrow), and basic facts (remember facts or use Touch Math if needed).
- SASH: For use with addition, helping students remember start in the ones column, add the numbers in each column, should I carry a numeral? and have I carried the correct numeral?
- MAMA: These letters stand for multiply the ones column, across to the tens column, multiply the bottom ones digit with the top tens digit, and add any number that was carried in step 2, for use with multi-digit multiplication problems.

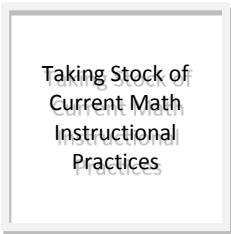
### Meta-Cognitive Strategy Instruction

Thinking about the thought processes involved in solving problems is the basis of meta-cognitive strategies. Students who have difficulty reading and understanding a problem, identifying the important information, representing that information, and developing a plan to solve a problem often have difficulty with meta-cognition (Montague & Jitendra, 2006).

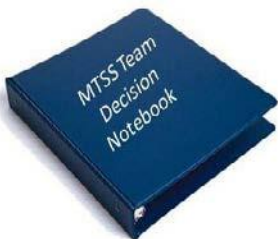
- Self-Monitoring (Self-Regulating) Strategies: Self-monitoring strategies keep students tuned in to the task at hand, helping students solve multi-step problems. Typically, older students use self-monitoring strategies when applying basic facts and concepts (Zrebiec, Mastropieri, & Scruggs, 2004). Included within self-monitoring strategies are self-instruction (talking oneself through the steps), self-questioning (using questions to cue certain steps, processes, and behaviors) and self-checking (discussed below). An example of how to approach self-monitoring instruction, found in Resource Math Strategies, discusses a checklist process to use when teaching students to use cognitive strategies. Once the steps are mastered, students are given a mnemonic device for remembering the steps. Examples are given for subtraction, addition, and multiplication.
- Self-Checking: This meta-cognitive strategy is used to help students think about their work once the problem is solved (Montague & Jitendra, 2006). Students are taught to check that:

- They understand the problem.
  - The information matches the problem.
  - The schematic representation reflects the problem.
  - The plan makes sense.
  - All important information is used in the plan.
  - The steps are completed in the right order.
  - The answer is correct.
- **Structured Organizers:** These graphic representations of checklists provide students a way to organize the information needed to solve a word problem. Students fill in the blanks (or chart) after each prompt.

Supplemental and intense supports may be delivered by a variety of qualified staff members (e.g., classroom teacher, a specialized teacher or another interventionist who has been trained for specific interventions). This decision is made by the leadership team and should be well defined before the process begins. Careful selection and adequate training of the staff members who are to deliver supplemental and intensive interventions are especially important in mathematics. Many staff members who are not math specialists lack confidence in their knowledge of math and their own ability to provide math intervention. It is critical to carefully examine staff development needs when selecting instructors to provide math interventions.



At this point the leadership team needs to identify current instructional practices being used in the building, determine who is using them, and identify the practices to be discontinued or replaced due to the lack of evidence or need. The leadership team should also identify instructional practices that will continue to be supported and any new practices that are needed. The leadership team should then identify which staff will be using each instructional practice and document all instructional practices that will be supported on the Taking Stock of Current Math Instructional Practices Tool.



### **Professional Development for Instruction and Ensuring Fidelity**

It is imperative the leadership team plan for the challenging task of providing support to staff, in order for staff members to change instructional practices and fully support MTSS.



The selection of the instructional strategies/practices is the first step. These instructional practices and strategies should be recorded on the Taking Stock of Current Math Instructional Practices Tool in the Decision Notebook. The second step is to plan ongoing support of staff to implement the necessary practices. To achieve fidelity of implementation, staff members need initial training as well as ongoing coaching and support to use these practices effectively and efficiently.

The building should also have a process in place to formally monitor implementation of ALL instructional practices, including core instructional practices, Tier 2 instructional practices, and Tier 3 instructional practices. Through this monitoring process, response and support via coaching can be provided in a timely and encouraging manner.

*It is imperative the principal be thoroughly engaged in the process of determining and providing for professional development and a monitoring system to ensure fidelity to instruction within the building.*

Use the following steps in deciding how to support staff in the use of evidence-based instructional practices:

- Develop a plan to provide professional development to appropriate instructional staff (ESOL, Migrant, Title, SPED, paraprofessionals, etc.).
- Determine the key elements of instruction that need to be monitored for fidelity.
- Determine a method (e.g., walk-through, peer coaching) to monitor key elements for fidelity.
- Develop and implement a plan to provide training and coaching to instructional staff that need additional assistance in providing instruction as identified through monitoring. Monitor the plan for fidelity of implementation.

Classroom walk-throughs can help ensure the fidelity of instructional practices. An example of a classroom walk-through is to provide models teams can use to identify observable examples of desired instructional practices within math classrooms. The example walk-throughs can be used to create an observation form for monitoring other key elements for instructional fidelity not included in these examples. Three examples of a classroom walk-through can be found in the following resources:

Fidelity Checks:  
Walk-Throughs

- Using Data to Improve Student Learning: School Processes— Observing a Mathematics Classroom (Collaborative Center for Teaching and Learning, 2009).
- High Schools That Work (HSTW) Key Mathematics Indices (High Schools that Work, 2009).
- What Leaders Should Look for In Effective Mathematics Classrooms (Mathematical Sciences Education Board, 2009).

Supplemental and intense supports may be delivered through a variety of qualified staff members (e.g., classroom teacher, a specialized teacher or another interventionist who has been trained for specific interventions). Ongoing professional development must be provided to these individuals to ensure the use of appropriate instructional practices. The leadership team needs to review the Taking Stock of Current Math Instructional Practices Tool to remind the team which instructional practices were identified to be supported.



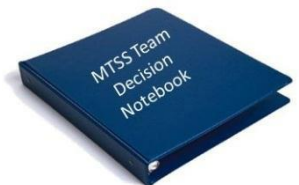
## Planning Professional Development

The building leadership team should identify the professional development needs related to the implementation of instructional strategies and practices by identifying which instructional practices will be utilized by which staff members, reviewing previously provided professional development, and identifying teachers needing additional support to implement the practices with fidelity.

<b>TEAM DISCUSSION</b>
In planning professional development it is helpful for the leadership team to consider these questions specific to each instructional strategy or practice:
1. Which staff members are expected to implement the strategy/practice?
2. Which staff will not be implementing the strategy/practice but will be providing support to students in settings where the use of the strategy/practice should be demonstrated?
3. Which staff members, if any, have experience with or have previously received professional development on the strategy/practice?
4. Which staff members need to attend initial professional development on the strategy/practice?
5. When (i.e., by which date) will staff first be expected to use the strategy/practice?
6. .When (i.e., by which date) will initial professional development be provided?
7. Who will provide the professional development?
8. Who will monitor the use/implementation (fidelity) of the strategy/practice?
9. What method will be used to monitor the use/implementation (fidelity) of the strategy/practice?

<b>TEAM DISCUSSION</b>
10. How frequently will the use/implementation (fidelity) of the strategy/practice be monitored?
11. When and how will ongoing professional development for staff be provided?
12. When and how will professional development for staff needing additional support to use the strategy/practice effectively be provided?
13. Who will provide professional development for new staff, and how will it be provided?

These questions are designed to help leadership teams as they begin the development of an overall Professional Development Plan. Once specific decisions are made, the building leadership team may record the results on the building's results-based staff development plan and/or on the Professional Development Planning Tool. Consider whether the discussion of professional development and fidelity of instruction has led to a need to develop an Action Plan or to add any items to the Stop-Doing List.



### **Review Policies and Practices for Instruction**

Now that the instructional practices plan has been completed, review district and building policies and practices regarding instruction. Identify whether there are policies and practices that need to be changed to align with the Instructional Practices Tool. Document these on the Policies & Practices Tool. Consider whether the discussion of policies and practices regarding instruction has led to a need to develop an Action Plan or to add any items to the Stop-Doing List.



<b>TEAM DISCUSSION</b>
1. Are there any policies (rules/guidelines) that require, prevent, or otherwise influence how, when, and what instructional strategies are used?

<b>TEAM DISCUSSION</b>
2. What are the practices (routines/traditions) that require, prevent, or otherwise influence how, when, and what instructional strategies are used?
3. Are there any practices that might belong on the Stop-Doing List?

Review the  
Communication  
Plan

Monitoring  
Fidelity of Paper  
Implementation

Model of  
Instruction

**Review the Communication Plan Related to Instruction**

Now that the Instructional Practices Tool has been finalized, review the plan for communication that needs to take place regarding instruction.

- Does the Communication Plan need to be modified?
- Are there any steps that need to be carried out to communicate decisions about instruction?
- Consider whether the discussion of a Communication Plan for instruction has led to the need to develop an Action Plan or to add any items to the Stop-Doing List.

**Intervention Structures: Model of Instruction**

The building leadership team must select a model for providing the tiered interventions needed for meeting the needs of students. A variety of possible models of instruction are available on the Comparison of Models chart, Appendix B. Keep in mind that the culture and logistics specific to a building will influence the implementation of any of the described models or the team’s creation of a model that is unique to the building.

**Core**

Core instruction provided to all students in the building should be consistent with research-based practices and the district allocation of instructional minutes. Core instruction in math should occur for a minimum of 50 minutes daily for kindergarten and first grade and a minimum of 50 to 60 minutes daily for grades 2-12. The time allotted for core must be sufficient to include differentiated instruction. When considering the model of instruction, remember that differentiation can be accomplished by providing extra assistance in core or at other times during the school day by planning peer groupings within the classroom or by creating varied class configurations as options within the school schedule.

## Supplemental

The model of instruction selected must provide instructional time for supplemental math interventions. This targeted small-group instruction typically includes flexible groupings of a maximum of three to five students for kindergarten and first grade and groupings of six to eight students for grades 2-12. Supplemental instruction provides a minimum of 20 to 30 minutes of intervention time 4 or 5 days a week in addition to core for all grade levels.

Supplemental intervention consists of explicit, direct, teacher-directed instruction and must allow sufficient time for use of more models and demonstrations, teaching of explicit problem-solving strategies, focused instruction on deficit areas, increased use of think alouds for illustrating thinking processes and checking for understanding, and increased corrective feedback and guided practice.

## Intensive

The difference within the intensive tier of instruction is the number of students in each group is much smaller; the instruction is more explicit, structured, and systematic; and the time in instruction beyond the core is increased. The group size for intensive instruction should be no larger than three students for kindergarten and first grade and no larger than five students for grades 2-12. Intensive instruction should be provided daily for 50 minutes for kindergarten and first grade and for 60 minutes for grades 2-12 in addition to core. Intensive instruction can be chunked to allow for more flexibility in scheduling. Whatever the model of support, the fluidity of grouping becomes critical to ensure students return to less intensive support as quickly as possible to reduce loss of instructional time in other settings. For the extreme few students who have severe and pervasive skill deficits, the extent of intensive instructional support required may result in a decision to use an alternative program to replace the general education curriculum and instruction. If a replacement program is used, intensive instruction would need to be provided 5 days a week for 60 to 90 minutes per day (Gersten et al., 2009b).

Before selecting a model of instruction, teams should look at universal screening data, if available, to have a rough estimate of the number of students who may need intervention. Generally speaking, data from assessments other than the universal screeners described in this guide may lead to insufficient intervention support. Consult the models in the Tiered System of Support Comparison of Models chart and discuss the pros and cons of each model. Then select a model of support that appears to be appropriate for the number of students in the building who might need intervention and that aligns with the building's Core Beliefs.



### Resource

A Comparison of Models of Instruction can be found in the Appendix.



*Principals must develop and implement schedules that support and protect the research findings related to appropriate instructional time.*

Considerations for Scheduling

Decision Rules for School Schedules

### Intervention Structures: Scheduling

When creating the schedule and selecting a model of instruction, it is prudent to first ensure classrooms are receiving adequate time for core instruction (minimum of 50 to 60 minutes). Then ensure sufficient time is being considered for supplemental and intense intervention for math. Building leadership teams may need to review decision rules about providing services to students who need interventions for both reading and math given the challenges of scheduling intervention time and the staff who provide these interventions. One way that math instruction scheduling is different from reading is that work on basic math skills/concepts can be effective in smaller, more frequent chunks of time. Time set aside for math core and tiered instruction does not necessarily need to be continuous.

### Scheduling for Early Grades (K-3)

The following example will show how a building can create a schedule to make the “Walk to Intervention” instructional model work. Simply put, this approach preserves a block of time at each grade level (K-3) for core instruction (50-60 minutes for math) and supplemental intervention (20-30 minutes for math) in these content areas. No “special” classes would be scheduled at this time, and all teachers and instructional aides would be part of the supplemental intervention.

Simplified K-2 Grade Schedule: Walk to Intervention

Time	Kindergarten			First Grade			Second Grade		
	Teach 1	Teach 2	Teach 3	Teach 1	Teacher 2	Teacher 3	Teacher 1	Teacher2	Teacher 3
8:30	Supplemental Math			Core Reading			Supplemental Reading		
9:00	Core Reading						Supplemental Reading		
9:30				Library			Supplemental Math		
10:00	Library						Supplemental Math		
10:30	Supplemental Reading			Lunch And Recess			Lunch And Recess		
11:00	Lunch and Recess			Lunch And recess			Library		
11:30	Library			Core Math			Library		
12:00	Core Math			Library			Supplemental Math		
12:30	Music/PE	Library	Music/PE	Music/PE			Library		
1:00	Library	Music/PE	Library	Library			Core Math		
1:30	Library	Music/PE	Library	Library			Core Math		
2:00	Library	Music/PE	Library	Library			Core Math		
2:30	Library	Music/PE	Library	Library			Core Math		
3:00	Library	Music/PE	Library	Library			Core Math		

**Resource**  
 Additional example schedules can be downloaded from the Kansas MTSS website in the Math Structuring Resource.

Students who would be best served by a particular specialist should be assigned to those professionals during instructional grouping. In some buildings, an enrichment teacher or media specialist also works with classes during this intervention time to ensure that students with advanced learning needs receive enrichment and extension opportunities. In this example schedule, every class at each grade level has a consistent time each day, thereby allowing structure and predictability. Many buildings find this type of schedule results in improved student behavior as well as enhanced academic

achievement. This type of scheduling requires planning and flexibility so that students can move in and out of instructional groups when needed as data dictates.

### **Scheduling for Intermediate and Secondary Grades (4-12)**

Scheduling for students in the intermediate grades (4-6) may look very different depending on whether or not these grades use a departmentalized structure. If the building does not use a departmentalized structure at those grade levels, then the structuring models and schedule will look more like those described for K-3. However, in buildings that have departmentalized intermediate grades, the model and schedule issues will be more like those for secondary buildings.

Planning the schedule for middle and high schools may be more complex because of increased numbers of students and the time for required and elective courses. NCA, NCAA and state graduation requirements will impact the schedule for core content courses at the high school level. Despite these limitations, core instruction provided to adolescent students' needs to include adequate time for the provision of differentiated instruction.

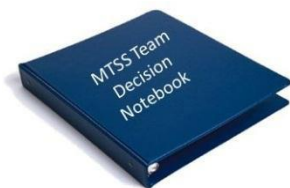
### **Supplemental Supports**

- Time for supplemental supports should be built into the master schedule and not infringe on time for recess or other content area instruction.
- An additional 20 to 30 minutes of instruction beyond the core for 4 or 5 days per week should be provided for supplemental instruction for math (Gersten et al., 2009a).
- Because supplemental instruction should complement core instruction, leadership teams should include collaborative planning time within the schedule.

### **Intensive Supports**

- Time for intensive supports should be built into the master schedule, although providing the amount of time needed for intensive instruction may not be possible without infringing on other allocated time in the schedule.
- The fluidity of grouping becomes critical to ensure that students can move to less intensive supports as quickly as possible to reduce loss of other instructional time.
- Intensive support for mathematics should consist of 60 additional minutes of instruction daily beyond the core instruction. These 60 minutes can be provided in time blocks best for the student (e.g., two 30-minute blocks).

The leadership team needs to complete each of the following items and include them in the Decision Notebook.



1. Identify where the time needed for core, supplemental, and intense instruction will fit within the schedule.
2. Identify staff to provide needed instruction.
3. Develop a detailed schedule for core, supplemental, and intense instruction.

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## Appendix A: Potential Math Intervention Curricula

Listed below are some of the interventions that have been independently researched, show promise, or are being used successfully in Kansas schools implementing MTSS. This is by no means an all-inclusive list. In an attempt to help schools be savvy consumers, the headings of each column are features which are considered “best practices” in curricula selection and can be used by schools to evaluate their current materials or potential Tier 2 and 3 curricula. An asterisk (\*) indicates a comprehensive curricula.

<u>Name</u>	<u>Grade Level</u>	<u>Most Appropriate Tier Use</u>	<u>Research Base or Promising Practice</u>	<u>Placement Test</u>	<u>Currently used in Kansas by MTSS school</u>	<u>Publisher Information</u>	<u>Technology based?</u>
Algebra Ready	6-10	Tier 2 & 3	Promising practice	Yes		Sopris West	No
Computation of Integers	5-Algebra	Tier 2	Research-based	Yes	Yes	Pearson	No
*Do the Math	1-6	Tier 2	Promising practice	Yes	Yes	Scholastic	No
Fastt Math (fact fluency)	2-12	Tier 2 or 3	Promising practice	Yes-adaptive	Yes	Scholastic	Yes
Fraction Nation	MS-Algebra	Tier 2	Research-based	Yes-adaptive		Scholastic	yes
Go Solve (schema based PS)	3-8	Tier 2	Promising practice	Yes-adaptive		Scholastic	Yes
*Inside Algebra	8-12	Tier 2	Promising practice	Yes		Voyager	Both
*Inside math	5-8	Tier 2	Promising practice	Yes-end of book test could be used	Yes	Sopris West	No
Math Facts In a Flash	3-12	Tier 2 or 3	Research-supported (Assessment	Adaptive	Yes	Renaissance Learning	Yes



<u>Name</u>	<u>Grade Level</u>	<u>Most Appropriate Tier Use</u>	<u>Research Base or Promising Practice</u>	<u>Placement Test</u>	<u>Currently used in Kansas by MTSS school</u>	<u>Publisher Information</u>	<u>Technology based?</u>
			for Effective Intervention)				
*Number Worlds	PreK-8	Tier 3 (being used with some Tier 2 in KS)	Research-supported (Center on Instruction)	Yes	Yes	SRA	No
Peer Assisted Learning Strategies (PALS)	K-6	Tier 2	Research-supported (study completed by Gersten et al.)	No, by grade level		Vanderbilt	No
Pirate Math	2-3	Tier 2	Research-supported	No		Vanderbilt	No
Skillbuilder	1-8	Tier 3	Research-based	Yes		SRA	No
Solving Equations	MS-algebra	Tier 2	Research-supported (IES)	Yes	Yes	Pearson	No
TransMath	5-10	Tier 2	Research-supported (NMP 2008 report)	Yes	Yes	Sopris West	No
*VMath	2-8	Tier 2	Research-based	Yes		Voyager	Both
*Voyager Math	2-8	Tier 2	Research-based	Yes	Yes	Cambium	No

## Appendix B: Comparison of Models

Model	Considerations	Advantages	Disadvantages	Scheduling	Resources
Pull Out	<ul style="list-style-type: none"> <li>• Works best when numbers of students needing assistance is small and/or done cross grade level.</li> <li>• Students in group need to have same instructional needs.</li> </ul>	<ul style="list-style-type: none"> <li>• Most similar to traditional practice.</li> <li>• Minimal logistical planning needed.</li> </ul>	<ul style="list-style-type: none"> <li>• Transition time to resource needed.</li> <li>• Most schools have more students to serve than this model accommodates.</li> <li>• Coordination with planning and reviewing progress monitoring data between teachers needed.</li> <li>• General education teachers need to make sure students being pulled out are not missing core curriculum.</li> </ul>	<ul style="list-style-type: none"> <li>• Typically, each grade level receives support ½ hr. to one hour each day.</li> <li>• Need to insure that students served with this model are not pulled out of general education curriculum.</li> </ul>	<ul style="list-style-type: none"> <li>• This model rarely requires extra or change in resources.</li> </ul>
In Class	<ul style="list-style-type: none"> <li>• Works best when numbers of students needing assistance is small.</li> <li>• Students in group need to have same instructional needs.</li> </ul>	<ul style="list-style-type: none"> <li>• Students stay in class for intervention time.</li> <li>• Classroom teacher is able to work with at least one group of his/her own students.</li> <li>• Students may be moved more flexibly in and out of intervention time.</li> </ul>	<ul style="list-style-type: none"> <li>• Most schools have more students to serve than this model accommodates.</li> <li>• Coordination with planning and reviewing progress monitoring data between other teachers who help is needed.</li> </ul>	<ul style="list-style-type: none"> <li>• Typically, each grade level receives support ½ hr. each day.</li> <li>• Can be done while other students are rotating through centers.</li> </ul>	<ul style="list-style-type: none"> <li>• Classroom supervisor may be necessary to protect uninterrupted intervention time.</li> </ul>

<b>Model</b>	<b>Considerations</b>	<b>Advantages</b>	<b>Disadvantages</b>	<b>Scheduling</b>	<b>Resources</b>
Intervention Team	<ul style="list-style-type: none"> <li>• Most likely used when number of students needing intervention is large, or beyond what can be done by the teacher and one support staff.</li> </ul>	<ul style="list-style-type: none"> <li>• A team can accommodate a larger number of groups.</li> <li>• Larger numbers of groups can make for more options when student’s needs change.</li> <li>• Allows time for additional support for Tier 3.</li> </ul>	<ul style="list-style-type: none"> <li>• Transition time to new groups needed.</li> <li>• General education teacher disconnected from student and instructional planning.</li> <li>• Interventionists report wanting to have the students for longer periods of time.</li> <li>• Training and support needs to be coordinated.</li> <li>• May be easy to overlook need to make core curricular changes.</li> </ul>	<ul style="list-style-type: none"> <li>• Typically, each grade level receives support ½ hr. each day.</li> </ul>	<ul style="list-style-type: none"> <li>• Depending on the number of intervention groups necessary, resources may need to be rethought in the school.</li> <li>• Make sure adequate training and support is built into the model.</li> <li>• Make sure students most in need have the most qualified interventionists.</li> </ul>
Walk to Intervention Cross-Class	<ul style="list-style-type: none"> <li>• Similar to intervention team approach, but grade-level teachers used as interventionists.</li> </ul>	<ul style="list-style-type: none"> <li>• Designated time by grade level insures that all students receiving extra reading time without conflicts to missing general education curriculum.</li> <li>• Allows for several certified staff to be providing reading interventions.</li> <li>• Easier to develop intervention groups for students needing enrichment</li> <li>• When teachers have built in collaborative time, discussions about groupings and individual students can be built in.</li> <li>• Allows time for additional support for Tier 3.</li> </ul>	<ul style="list-style-type: none"> <li>• Transition time to new groups needed.</li> <li>• General education teacher sometimes disconnected from student and instructional planning.</li> </ul>	<ul style="list-style-type: none"> <li>• Each grade level coordinates intervention time with other reading teachers (reading specialists/ special education)</li> </ul>	<ul style="list-style-type: none"> <li>• Depending on the number of intervention groups necessary, teachers may be able to provide more guided assistance to students barely on track. On the other hand, other building or district personnel may be called upon to assist.</li> </ul>

<b>Model</b>	<b>Considerations</b>	<b>Advantages</b>	<b>Disadvantages</b>	<b>Scheduling</b>	<b>Resources</b>
Walk to Intervention Cross-Grade	<ul style="list-style-type: none"> <li>Consider when the number of students on track is considerably less than those not on track.</li> </ul>	<ul style="list-style-type: none"> <li>Allows for more individualized and intense instruction based on reading and skill level.</li> <li>Focus on reading increased due to no transition time necessary.</li> <li>Teacher provided time to know student's skill level and increased time allows him/her more flexibility in meeting needs.</li> </ul>	<ul style="list-style-type: none"> <li>Requires difficult decisions to be made regarding other important curriculum matters.</li> <li>Requires thinking about things very differently.</li> </ul>	<ul style="list-style-type: none"> <li>Scheduling takes into consideration resources needed and grade level requirements.</li> </ul>	<ul style="list-style-type: none"> <li>Resources can be allocated in larger chunks of time.</li> </ul>
Alternative Class (Required Elective)	<ul style="list-style-type: none"> <li>Students with similar needs are scheduled with an intervention teacher for basic skills instruction, while remaining in the core English/Language Arts (ELA) or math course.</li> </ul>	<ul style="list-style-type: none"> <li>Works well in high school schedule.</li> <li>Enables students to progress in core content classes while improving basic literacy or math skills.</li> <li>The interventionist may be able to provide both student instruction and teacher consultation.</li> <li>Convenient for using purchased curriculum for struggling readers.</li> </ul>	<ul style="list-style-type: none"> <li>Students lose the choice of what may be a preferred elective class.</li> <li>Requires having a staff member with specialized knowledge of basic skills instruction.</li> </ul>	<ul style="list-style-type: none"> <li>Requires that students with common needs be available during the same class period.</li> </ul>	<ul style="list-style-type: none"> <li>The number of students and their needs will determine how many class periods the interventionist needs to schedule</li> </ul>
Intervention Team (Homeroom)	<ul style="list-style-type: none"> <li>Each teacher takes a group of students for intervention, including students at benchmark or above.</li> </ul>	<ul style="list-style-type: none"> <li>Works well in middle school schedules.</li> <li>Providing intervention during homeroom time helps with fluidity of grouping.</li> </ul>	<ul style="list-style-type: none"> <li>Requires common planning time for teachers to collaborate.</li> </ul>	<ul style="list-style-type: none"> <li>Instructional groups can be matched to teachers' individual skills.</li> </ul>	<ul style="list-style-type: none"> <li>Some buildings may need to increase the amount of time allowed for homeroom</li> </ul>

Model	Considerations	Advantages	Disadvantages	Scheduling	Resources
All School Seminar or Advisory Period	<ul style="list-style-type: none"> <li>All students receive extensions, additional practice, or supplemental or intense instruction during seminar time.</li> </ul>	<ul style="list-style-type: none"> <li>Many secondary schools already have an advisory or seminar period built into their schedules.</li> <li>Assures that all students (advanced learners, benchmark students, and students with learning difficulties) receive some type of intervention.</li> <li>Enables departmental planning for interventions.</li> </ul>	<ul style="list-style-type: none"> <li>Requires that focus of seminar be changed to instruction. This may mean a loss of time for student organizations and may also conflict with scheduled teacher planning times.</li> </ul>	<ul style="list-style-type: none"> <li>The way students are scheduled into seminar may need to be reorganized.</li> </ul>	<ul style="list-style-type: none"> <li>Changed purpose of seminar will require that more teachers are engaged in instruction during that period.</li> </ul>

Option 1	Option 2	Option 3	Option 4
<ul style="list-style-type: none"> <li>All English/Language Arts (ELA) classes are scheduled throughout the school day and are heterogeneously grouped.</li> <li>A reading support elective (mandatory) is added to the schedule to allow for enrichment for Tier 1 or Tier 2 intervention.</li> <li>Students in need of Tier 3 intervention receive 2 periods of intense instruction in addition to the ELA class.</li> <li>Intervention classes are blended across grades and populations based on student need.</li> <li>Tier 2 and Tier 3 intervention classes are scheduled during the same period as much as possible.</li> </ul>	<ul style="list-style-type: none"> <li>ELA classes are scheduled throughout the day.</li> <li>ELA classes are heterogeneously grouped.</li> <li>Students are pulled out for Tier 2 or Tier 3 intervention during other classes (one period for Tier 2 and two periods for Tier 3).</li> <li>Tier 2 intervention may occur within another class (e.g., social studies).</li> <li>Intervention classes are homogeneously grouped based on student need.</li> <li>Intervention classes are blended across grades and populations.</li> </ul>	<ul style="list-style-type: none"> <li>ELA classes are double blocked (one period core credit and one period elective).</li> <li>ELA classes are scheduled at the same time of the day as much as possible.</li> <li>ELA classes are homogeneously grouped based on assessed need and grade level.</li> <li>Pacing, intensity, content, exposure to the core and explicit instruction are based on assessed student need.</li> <li>Classes are blended across populations.</li> <li>This option is useful when large numbers of students need intervention.</li> </ul>	<ul style="list-style-type: none"> <li>ELA classes are heterogeneously grouped for students in Tier 1 and Tier 2.</li> <li>ELA classes are scheduled throughout the day.</li> <li>Students requiring Tier 3 intervention are removed from grade level curriculum and receive 2 blocked periods of intense intervention. The class counts for one grade level and one elective class. Classes are blended across grade levels and populations.</li> <li>Tier 2 classes are homogeneously grouped and replace one elective class. Classes are blended across grade levels and populations.</li> <li>Tier 2 and Tier 3 classes are parallel scheduled as much as possible.</li> </ul>

Table above: Allain, J. (2009, September). *Precise planning for RtI for middle/high Schools: Finding your best fit*. Presentation at the Kansas MTSS Symposium, Wichita, KS.

## Appendix C: The Importance of Fluency

According to the National Mathematics Advisory Panel (2008), the ability to recall basic mathematics facts is critical for general success in mathematics. The National Mathematics Advisory Panel (2008), Common Core State Standards Initiative (2010), and National Council of Teachers of Mathematics (2006) all state that the quick and accurate recall of math facts is a core skill and prerequisite for higher level learning.

Hasselbring (2012) states “when students lack fluency in the foundational skills, performance requiring application of those skills is likely to be painfully slow, difficult, and full of errors.” Students must develop *computational fluency* if they are to solve complex and interesting problems (*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* National Council of Teachers of Mathematics, 2006). Fluency frees up working memory for higher order applications rather than creating cognitive overload with the mechanics of performance.

Vos (2009) outlined a three-stage progression in the ability to recall basic math facts from “concept development” to “practice” (with a recall rate of 3–4 seconds per fact) and finally to “automaticity,” defined as a recall rate of 2 to 3 seconds per fact. Fluency standards accepted by most schools range from 40 to 60 digits correct per minute. When these standards are met it is generally considered that the student has reached fluency (Hasselbring, 2012). Hasselbring (2012) notes three concerns with using this measure of fluency.

1. Students develop such rapid “counting” strategies that they can still meet the criteria for fluency and not have developed declarative knowledge of the facts.
2. A per-second, rate fails to identify those facts that are part of the declarative knowledge network and those that are answered using counting strategies.
3. More than half of the facts in all operations have a 0, 1, or 2 as part of the fact set.

Given these concerns, Hasselbring (2012) goes on to point out that the use of rate, which is the number of digits correct per minute, to measure mathematical fluency creates a “false positive.” This can happen because the teacher has no idea how long the student took to perform each individual fact. The student may have taken 5 seconds to perform one fact and still be able to meet the 40 to 60 digits correct per minute and appear to be fluent with all of the facts presented.

An alternative to **rate per minute** as a measure of fluency is the use of **chronometric analysis**. Chronometric analysis requires that response latency be measured for each fact with the benchmark set at 0.8 seconds per fact. The response latencies can then be used to determine precisely which facts fall above and below the criterion for fluency. However, to accurately measure response latency a computer must be used. An example of such a computerized program is Fastt Math from Scholastic.

Automatic recall must be developed over time with sufficient instruction, practice, and feedback (Baroody, 1999; Willingham, 2009). VanDerHeyden and Burns (2008) described this process as moving from acquisition (accuracy) to proficiency (speed). Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts (Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools, IES, 2009).

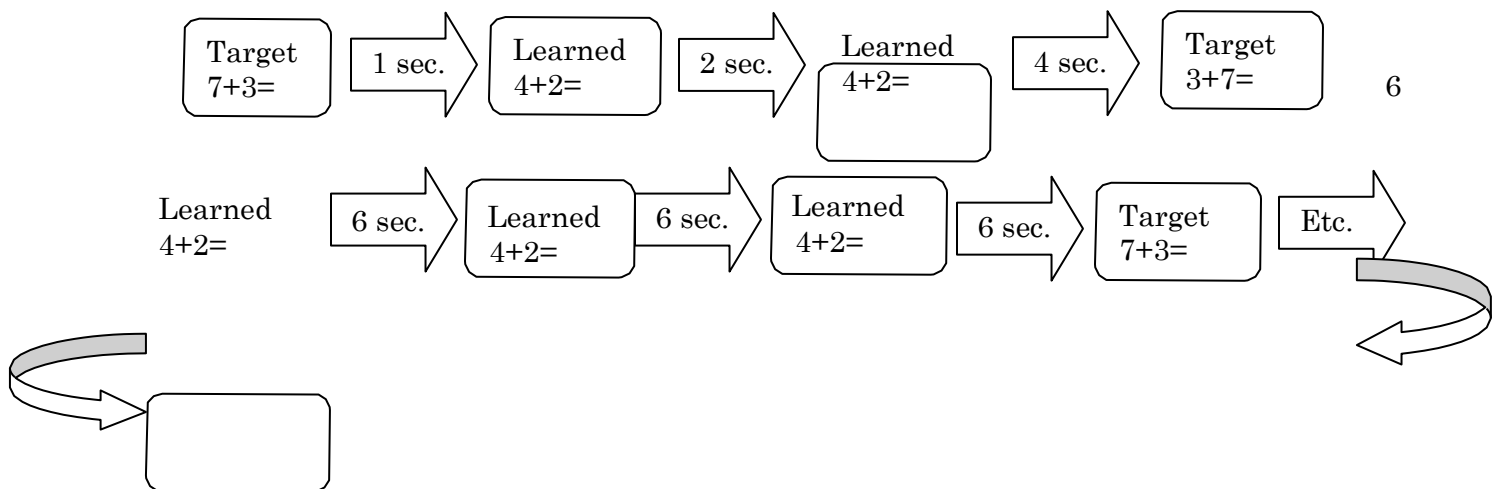
Before fluency can be developed, information must be moved from working memory to long-term memory. This is accomplished through deliberate practice. Examples of deliberate practice include:

- Targeted practice in which one is engaged in developmentally appropriate activities.
- Real-time corrective feedback that is based on one's performance.
- Intensive practice on a daily basis that provides results that monitor current ability.
- Distributed practice that provides appropriate activities over a long period of time.
- Self-directed practice for those times when a coach, mentor, or teacher is not available.

The instruction in fluency should include the following components.

- Guided practice on new information.
- Controlled response time.
- Corrective feedback.
- Spaced presentation of new information using expanding recall.

The spaced presentation of new information using expanding recall is performed by giving the student facts which they are working on for fluency, also known as target facts, with several seconds of time separating them. During those time spaces the student should receive facts they already have fluency with, also known as learned facts. So fluency work with  $7+3=?$  and  $3+7=?$  as the target fact would look like the following example:



The table below lists the empirically derived levels of instruction found for fluency for grades 2<sup>nd</sup>/3<sup>rd</sup> and 4<sup>th</sup>/5<sup>th</sup> (Burns, et.al., 2006)

<b>Level of Instruction</b>	<b>2<sup>nd</sup>/3<sup>rd</sup> Grade</b>	<b>4<sup>th</sup>/5<sup>th</sup> Grade</b>
<b>Frustrational</b>	0 – 13 Digits Correct Per Minute	0 – 23 Digits Correct Per Minute
<b>Instructional</b>	14 – 31 Digits Correct Per Minute	24 – 49 Digits Correct Per Minute
<b>Mastery</b>	> 31 Digits Corrects Per Minute	> 49 Digits Correct Per Minute

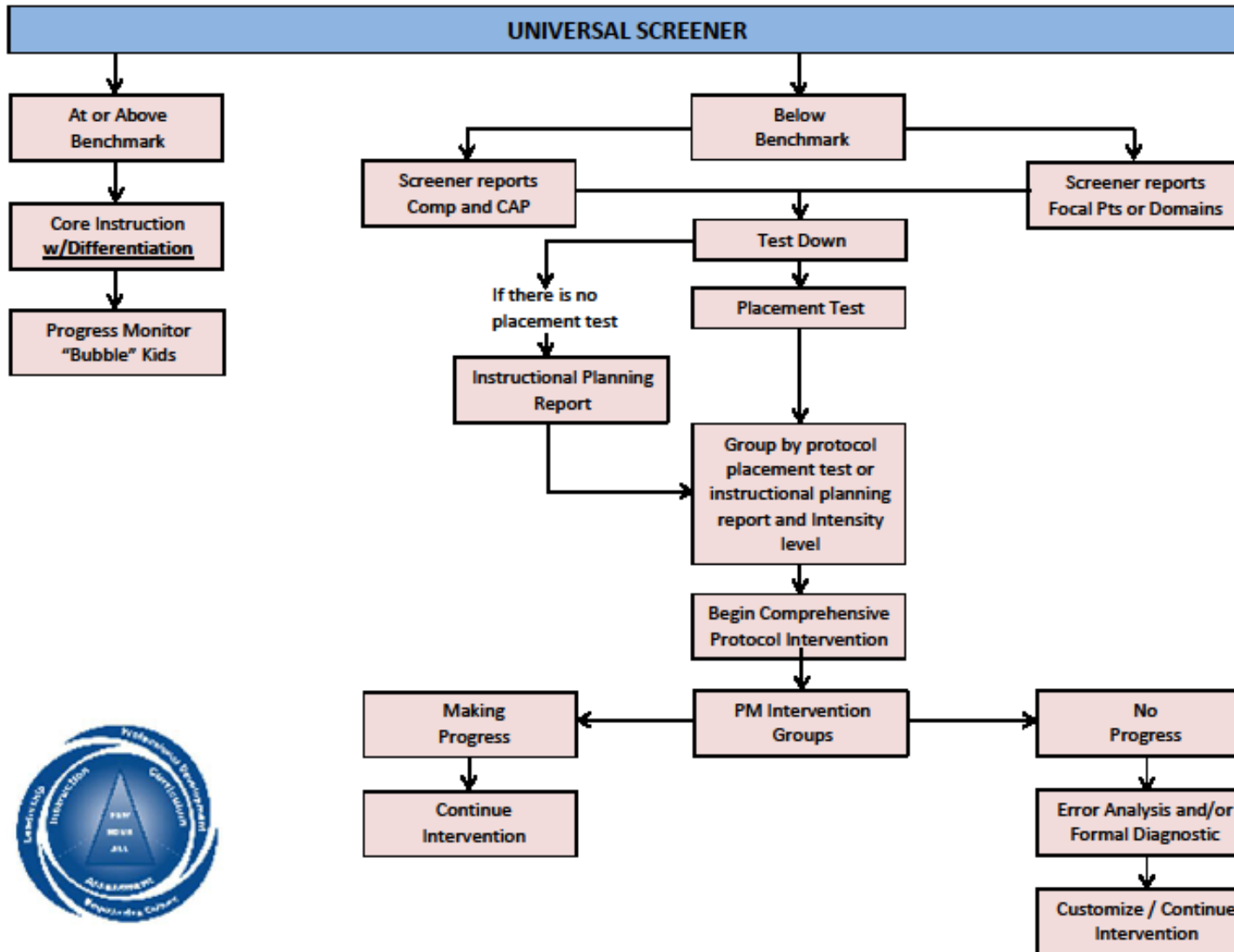
If a chronometric analysis is being performed to determine fluency, the benchmark is 0.7 seconds per fact. Once fluency reaches a particular level of accuracy and precision, it can be maintained at that level over long periods of time with only a small amount of practice from time to time (Hasselbring, 2012).



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## MTSS MATH GROUPING PROCESS



## MTSS Math Glossary of Terms

Last updated 3/20/2012

Term	Definition
<b>Adaptive Instruction</b>	Instruction that changes to meet the changing needs of students.
<b>Adaptive Expertise</b>	The ability to apply a number of strategies for a particular operation, or a number of procedures, flexibly and creatively to solve problems.
<b>Chunking</b>	Breaking information into smaller, more manageable sets of skill- or concept-related groupings. This practice is especially helpful for students with limited working memory.
<b>Computation</b>	An assessment of math computational skills, ranging from basic facts and simple arithmetic to complex calculations, including equations, at upper grade levels.
<b>Concepts/Application</b>	An assessment of general mathematics problem-solving skills which includes math concepts and math vocabulary.
<b>Concrete-Representational-Abstract Techniques</b>	A three-part instructional strategy where the teacher uses concrete materials (manipulatives) to model a concept to be learned, then uses representational terms (pictures, graphs, number lines, ten frames), and finally uses abstract, symbolic terms (numbers, math symbols).
<b>Dyscalculia</b>	A form of learning disability that causes an individual to have difficulty in understanding concepts of quantity, time, place, value, and sequencing, and in successfully manipulating numbers or their representations in mathematical operations.
<b>Magnitude Comparison</b>	The ability to discern quickly which number is the greatest in a set, and to be able to weigh relative differences in magnitude efficiently.
<b>Mathematical Proficiency</b>	A term developed by the National Research Council that describes five interrelated strands of knowledge, skills, abilities, and beliefs that allow for mathematics manipulation and achievement across all mathematical domains (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) (Kilpatrick et al, 2001).
<b>Missing Number</b>	An assessment of strategic counting, where a series of two numbers and a blank are presented to a student and the student fills in the blank. The blank might occur in the initial, middle, or final position of the series of three, for example: ____, 16, 17 or 3, __, 5 or 11, 12, ____.
<b>Open Questions</b>	Broad-based questions with many appropriate responses or approaches; the questions serve as a vehicle for differentiating instruction if the range of possibilities allows students at different developmental levels to succeed even while responding differently.
<b>Operations</b>	Mathematical procedure such as addition, subtraction, multiplication, and division.
<b>Parallel Tasks</b>	Parallel tasks are sets of two or more related tasks or activities that explore the same big idea but are designed to suit the needs of students at different developmental levels, but that get at the same big idea and are close enough in context that they can be discussed simultaneously.

<b>Proficiency</b>	The National Mathematics Advisory Panel (2008) defines proficiency as students who understand key concepts; achieve automaticity as appropriate; develop flexible, accurate, and automatic execution of standard algorithms; and use competencies to solve problems. “In a balanced approach, a mathematical program will develop proficiency with basic computational and procedural skills, promote conceptual understanding, and systematically move students toward being strategic problem solvers.”
<b>Quantity Discrimination</b>	An assessment of magnitude comparison. At the preschool level the student is asked to compare one group of circles with another group of circles and tell which group is larger. At the kindergarten and first grade level, the student is asked to tell which of two numbers is larger.
<b>Strategic Counting</b>	The ability to understand how to count efficiently and to use counting to solve problems. Includes knowledge of rudimentary counting principles and skills such as counting on.
<b>Subitizing</b>	The ability to determine the number of objects at a glance, without counting.

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