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Math in the Early Years

A Strong Predictor for Later School Success

The earliest years of a child's education—from birth through 3rd grade—set the foundation upon which future learning is built. In recent years, state policymakers have emphasized the need to improve children's reading skills early on because a lack in this essential skill is a strong predictor of low student performance and increased high school dropout rates. By 2012, a total of 32 states plus the District of Columbia had policies in statute aimed at improving 3rd-grade literacy, with 14 of those states requiring retention of students on the basis of reading proficiency. While the emphasis on reading proficiency is critical, research shows that the development of mathematics skills early on may be an even greater predictor of later school success. Early knowledge of math not only predicts later success in math, but also predicts later reading achievement even better than early reading skills.

Young children have a surprising capacity to learn substantial mathematics, but most children in the U.S. have a discouraging lack of opportunities to do so. Too many children not only start behind, but they also begin a negative and immutable trajectory in mathematics, with insidious long-term effects. These negative effects are in one of the most important subjects of academic life and also affect children's overall life course.

What's Inside

The good news is that programs and curricula designed to facilitate mathematical learning from the earlier years, continued through elementary school, have a strong positive effect on these children's lives for many years thereafter. Starting early—in preschool—with high-quality mathematics education, creates an opportunity for substantial mathematical learning in the primary years that builds on these foundational competencies.

This issue of *The Progress of Education Reform* reveals five surprising findings about the importance of early math learning, and provides implications and recommendations for state policy.

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Surprising Research Findings

Surprise 1: There is predictive power in early mathematics

Mathematical thinking is cognitively foundational[1], and children's early knowledge of math strongly predicts their later success in math.[2] More surprising is that preschool mathematics knowledge predicts achievement even into high school.[3] Most surprising is that it also predicts later reading achievement even better than early reading skills.[4] In fact, research shows that doing more mathematics increases oral language abilities, even when measured during the following school year. These include vocabulary, inference, independence, and grammatical complexity.[5] Given the importance of mathematics to academic success in all subjects[6], all children need a robust knowledge of mathematics in their earliest years.

> *Before her 4th birthday, Abby was given five train engines. She walked in one day with three of them. Her father said, "Where's the other ones?" "I lost them," she admitted. "How many are missing?" he asked. "I have one, two, three. So [pointing in the air] fooooour, fiiiive … two are missing, four and five. [pause] No! I want these to be [pointing at the three engines] one, three, and five. So, two and four are missing. Still two missing, but they're numbers two and four." Abby thought about counting and numbers—at least small numbers—abstractly. She could assign one, two, and three to the three engines, or one, three, and five! Moreover, she could count the numbers. That is, she applied counting … to counting numbers!*

Surprise 2: Given opportunities to learn, young children possess an informal knowledge of mathematics that is amazingly broad, complex, and sophisticated[7]

When children 'play,' they are often doing much more than that. Preschoolers can learn to invent solutions to solve simple arithmetic problems, and almost all of them engage in substantial amounts of pre-mathematical activity in their free play.[8] In fact, early childhood programs that include more mathematics have increased higher-level free play, all of which promotes self-regulation and executive function. Through higher-level play, children explore patterns, shapes, and spatial relations; compare magnitudes; and count objects. Importantly, this is shown to be true regardless of the children's income level or gender.[9] These explorations through play are pre-mathimatical. It is high-quality education that can help all children utilize their inherent skills in order to truly mathematize.[10] However, if high-quality mathematics education does not start in preschool and continue through the early years, most children are trapped in a trajectory of failure.[11]

Surprise 3: Teachers vastly underestimate what their children know and can learn[12]

In numerous countries, professionals in multiple educational roles vastly underestimate beginning students' abilities.[13] One study showed that groups of teachers, teacher trainers, and counselors who worked with preschoolers underestimated the mathematical competencies of these very same students when they entered kindergarten.[14] For example, more than 80% of the students could count out nine marbles, but the adults' estimates were from 20% to 50%. More than 40% of the students could subtract 10 – 8 without objects, but all adults estimated less than 10%. If teachers and those who work with teachers underestimate what students already know and can learn, they will not present appropriate, challenging mathematics activities.

Surprise 4: All students need a math intervention

Most children benefit from a math intervention.[15] As W. Steven Barnett and others' research has shown, it is not just the poorest children who need interventions.[16] When they enter kindergarten, most children are behind their peers from the best-funded communities. That is, there is a significant gap between every "quintile" and the highest 20% (see Figure 1 on following page). Still, those in poverty need mathematics interventions the most.[17] There is a three-year difference in mathematics developmental level for students from low-resource versus high-resource communities.[18]

POLL RESULTS

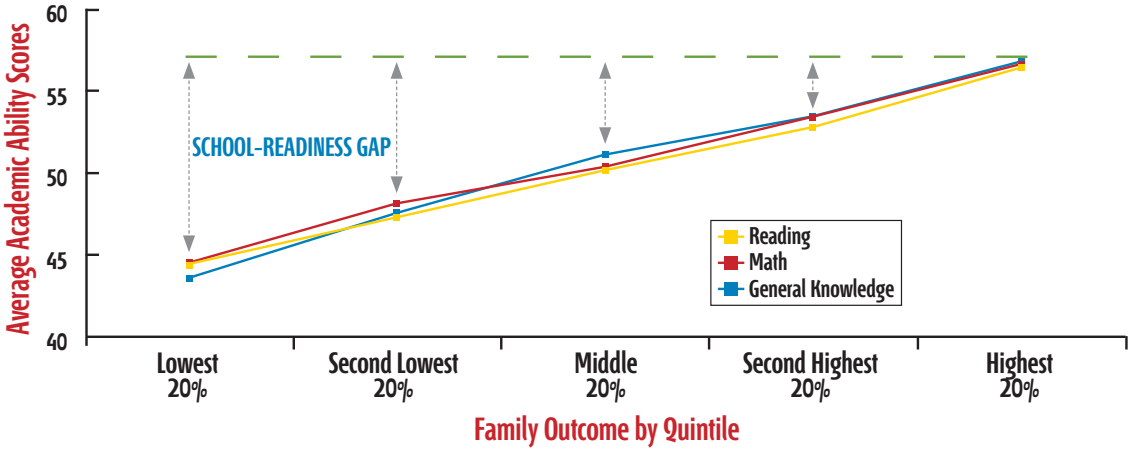
What do parents and children say about math?

98%	Math is very important (parents)
89%	Math is very important (children)
55%	I am good at math (children)
91%	Schools need to help the brightest learn math (parents)
55%	Children who like math before middle school

Source: Harrison Group, PROMISE research, Phase 2, June 2010, Michigan State University.

Figure 1: Closing the school-readiness gap

When they enter kindergarten, children from lower- and middle-income families are, on average, far behind their wealthier peers in reading, mathematics, and general knowledge. High-quality preschool could help close this gap in school readiness.

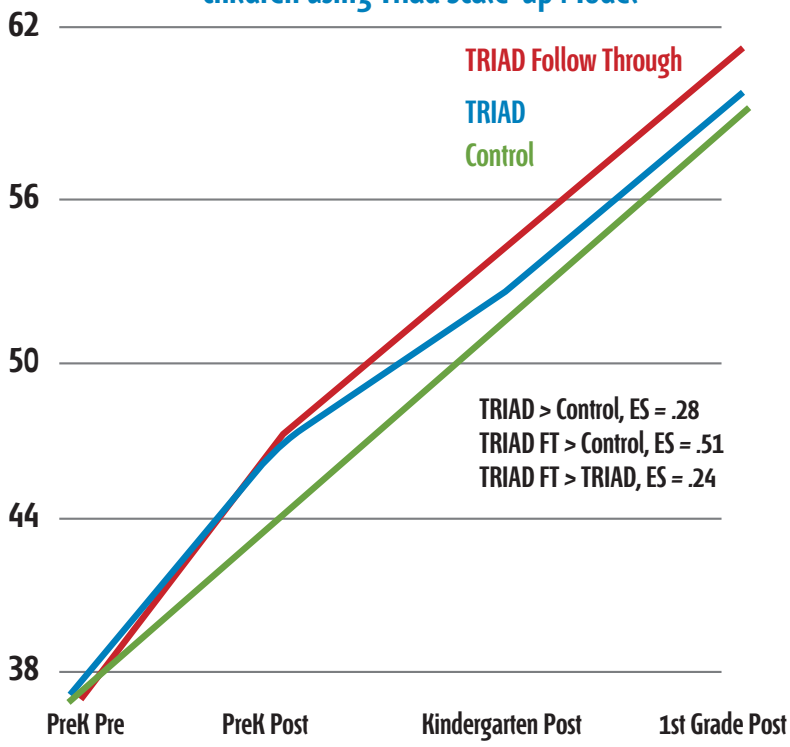


Source: Analysis of data from the *Early Childhood Longitudinal Study, Kindergarten Class of 1998–99* (See nces.ed.gov/ecls/kindergarten.asp) by W. Steven Barnett and Milagros Nores for the National Institute for Early Childhood Education Research.

Surprise 5: We know a lot

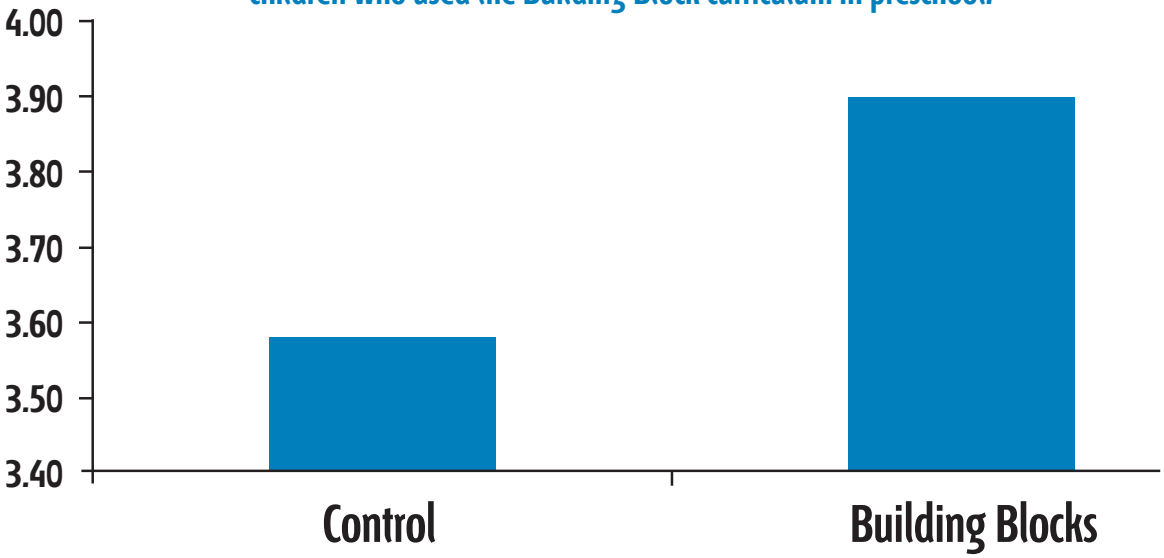
A lot is known about how children think about and learn math, and teachers can use learning trajectories to synthesize this knowledge into effective interventions for children. There are books and research available to districts that detail the learning trajectories that can help underlie scientific approaches to standards, assessment, curricula, and professional development and provide teachers with curricula that show effect sizes that are large and significant.[19] Two such models are the Building Blocks curriculum and TRIAD scale-up model (see figures 2 and 3). High-quality instruction has meaningful effects on children's mathematics knowledge.[20]

Figure 2: Mathematics achievement scores for children using Triad Scale-up Model



Source: D.H. Clements, J. Sarama, C.B. Wolfe, and M.E. Spitler, "Longitudinal Evaluation of a Scale-up Model for Teaching Mathematics with Trajectories and Technologies: Persistence of Effects in the Third Year," *American Educational Research Journal*, 50(4), (2013): 812–850, doi: 10.3102/0002831212469270.

Figure 3: Expressive oral language scores at the beginning of kindergarten for children who used the Building Block curriculum in preschool.



Source: J. Sarama, A. Lange, D.H. Clements, and C.B. Wolfe, "The Impacts of an Early Mathematics Curriculum on Emerging Literacy and Language," *Early Childhood Research Quarterly*, 27, (2012): 489–502, doi: 10.1016/j.ecresq.2011.12.002.

Policy Implications and Recommendations

The Importance of High-Quality Curriculum and Instruction

The quality of mathematics education varies across settings but is generally disappointing, especially in the earliest years. For example, 60% of 3-year-olds had no mathematical experience of any kind across 180 observations.[21] Even if a program adapts an ostensibly "complete" curriculum, mathematics is often inadequate, with the most commonly used engendering no more math instruction than a control group.[22] It is little surprise, then, that evaluations show little or no learning of mathematics in these schools.[23] As an example, observations of Opening the World of Learning (OWL), which includes mathematics in its curriculum, found that out of a 360-minute school day, only 58 *seconds* were devoted to mathematics. Most children made no gains in math skills, and some lost mathematics competence over the school year.[24] Teachers often believe that they are "doing mathematics" when they provide puzzles, blocks, and songs. Even when they teach mathematics, that content is usually not the main focus, but is "embedded" in a fine-motor or reading activity.[25] Unfortunately, evidence suggests such an approach is ineffective.[26] To ensure a program is truly effective, policymakers and school leaders must prioritize investing in high-quality math curricula and instruction that meet the needs of all students.

58% Percent of adults who cannot compute a 10% tip

71% Percent who cannot compute the interest paid on a loan

78% Percent who cannont calculate miles per gallon on a trip

Source: G.W. Phillips, *Chance Favors the Prepared Mind: Mathematics and Science Indicators for Comparing States and Nations* (Washington, DC: American Institutes for Research, 2007).

Qualified Instructors

Teacher certification for pre-K through 3rd-grade teachers should emphasize both knowledge of the subject (specifically, a profound knowledge of the math taught in early and elementary years) and strengths in pedagogy. It is only recently that some states are requiring teachers to be evaluated on fluency in literacy instruction. What we now know is that math instruction is far more effective coming from a specialist who understands both the subject matter and the most effective ways in which young children learn math. A successful program will be one that ensures that early math instructors specialize in these areas. One solution may be for a school to designate a teacher in each grade who is responsible for teaching only math to all students.

Seamless Learning Trajectories

The most common argument offered for limiting investments in preschool is that the gains made are soon lost as a child matriculates through the early primary grades. The losses primarily signify a siloed approach to education, where each grade level and teacher holds different expectations for students, creating a learning trajectory that is not seamless. Therefore, in order for students to benefit from math instruction in the early years, primary grade teachers must build on early math interventions and engage students in more interesting, challenging, and substantial math lessons as students progress through competency levels. If there are follow-through interventions in kindergarten and the primary grades, students maintain their preschool advantages.[27] This effect is highlighted in Figure 2 (page 3), which presents a significant, positive effect on student math scores when the Triad Model is used on an ongoing basis.

Professional Development

Early math is not often emphasized in teacher preparation programs. As a result, pre-service and in-service teachers alike lack content knowledge, such as understanding of mathematical concepts and procedures. More importantly, they lack mathematics knowledge for teaching—how mathematical knowledge is interconnected and connected to the real world, how a student's thinking about mathematical content develops, and how mathematical content can be taught in a meaningful manner.[28] They suffer from negative effects, including math anxiety and a lack of confidence in their own mathematical ability and ability to teach mathematics—beliefs that lead to undervaluing the teaching of mathematics or prevent effective teaching.[29] Therefore, professional development for early childhood mathematics needs to address content (mathematical) knowledge, particularly mathematics knowledge for teaching, as well as pedagogical knowledge, and affective issues.[30]

Conclusion

It is time to begin shifting the mindset of teachers, district leaders, and policymakers from a 'reading only' early intervention strategy to one that incorporates and even emphasizes mathematical thinking and reasoning. To do so, stakeholders should take a deep look into the current state of early math instruction beginning in preschool and creating a seamless trajectory for math learning through the early grades. Education leaders should find ways to maximize children's abilities to learn by evaluating the current state of mathematics instruction within schools, based not only on the current curricula, but also the time committed to instruction, as well as who is doing that instructing. Most children can master the required skills early if given the chance.

Dr. Clements engages in math activites with two kindergarteners in order to help them understand the core unit of patterns.

Endnotes

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ECS Resources

Recent State Policies/Activities: Preschool Policies

Summaries and links to newly enrolled or enacted legislation and recently approved state board rules from across the states. Updated weekly.

http://www.ecs.org/ecs/ecscat.nsf/WebTopicViewAll?OpenView&Start=1&Count=1000&Expand=204#204

Third Grade Reading Policies

This paper outlines state policies relating to 3rd-grade reading proficiency, including identification of, intervention for, and retention of struggling readers in the P-3 grades. The paper provides a state-by-state policy summary, sample statutory language, and highlights from bills enacted this year.

http://www.ecs.org/clearinghouse/01/03/47/10347.pdf

ECS Research Studies Database:

Find research studies that provide features that define high-quality learning environments for PreK-3 students:
http://www.ecs.org/rs/SearchEngine/SearchResults.aspx?faq_id=a0870000004rrIvAAI
or on what mathematics practices impact student achievement:
http://www.ecs.org/rs/SearchEngine/SearchResults.aspx?faq_id=a0870000006yt5BAAQ.

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GE Foundation

The Education Commission of the States is a nationwide nonprofit organization formed in 1965 to help governors, state legislators, state education officials, and others to develop policies to improve the quality of education. ECS is the only nationwide, nonpartisan interstate compact devoted to education at all levels.

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Building Blocks of Early Mathematics

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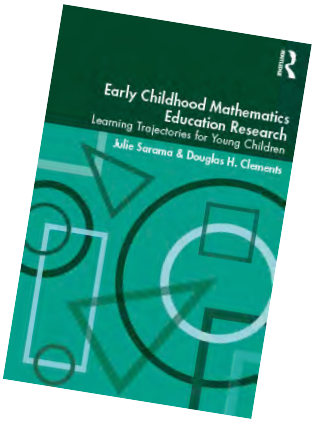
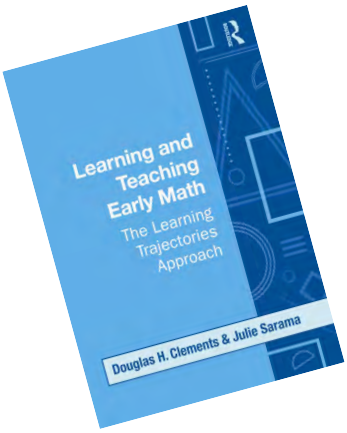
http://www.triadscaleup.org/





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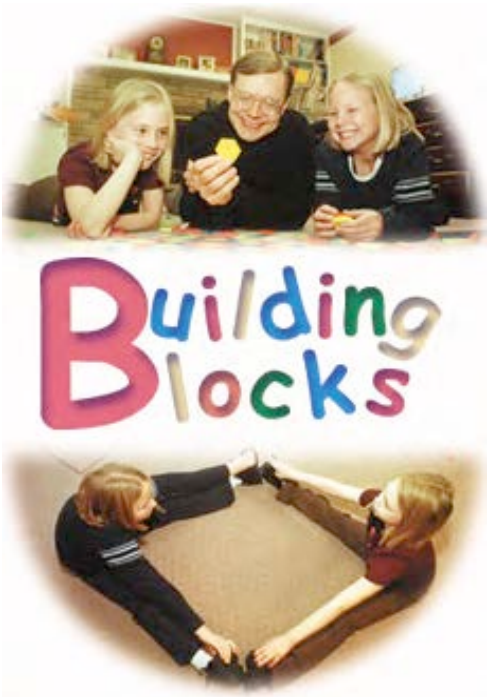

www.mheonline.com/earlychildhoodconnection/

http://www.buildingblocksmath.org/

for research, http://www.triadscaleup.org/

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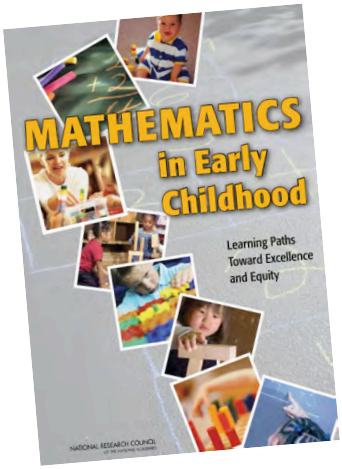

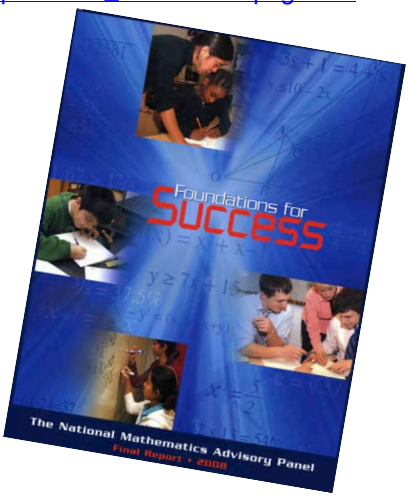

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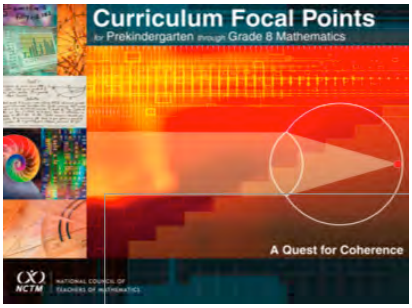


National Math Panel—http://www.ed.gov/

NCTM — www.nctm.org

Curriculum Focal Points and Connections for Prekindergarten

The set of three curriculum focal points and related connections for mathematics in prekindergarten follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Prekindergarten Curriculum Focal Points

Number and Operations: Developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison

Children develop an understanding of the meanings of whole numbers and recognize the number of objects in small groups without counting and by counting—the first and most basic mathematical algorithm. They understand that number words refer to quantity. They use one-to-one correspondence to solve problems by matching sets and comparing number amounts and in counting objects to 10 and beyond. They understand that the last word that they state in counting tells "how many," they count to determine number amounts and compare quantities (using language such as "more than" and "less than"), and they order sets by the number of objects in them.

Geometry: Identifying shapes and describing spatial relationships

Children develop spatial reasoning by working from two perspectives on space as they examine the shapes of objects and inspect their relative positions. They find shapes in their environments and describe them in their own words. They build pictures and designs by combining two- and three-dimensional shapes, and they solve such problems as deciding which piece will fit into a space in a puzzle. They discuss the relative positions of objects with vocabulary such as "above," "below," and "next to."

Measurement: Identifying measurable attributes and comparing objects by using these attributes

Children identify objects as "the same" or "different," and then "more" or "less," on the basis of attributes that they can measure. They identify measurable attributes such as length and weight and solve problems by making direct comparisons of objects on the basis of those attributes.

Connections to the Focal Points

Data Analysis: Children learn the foundations of data analysis by using objects' attributes that they have identified in relation to geometry and measurement (e.g., size, quantity, orientation, number of sides or vertices, color) for various purposes, such as describing, sorting, or comparing. For example, children sort geometric figures by shape, compare objects by weight ("heavier," "lighter"), or describe sets of objects by the number of objects in each set.

Number and Operations: Children use meanings of numbers to create strategies for solving problems and responding to practical situations, such as getting just enough napkins for a group, or mathematical situations, such as determining that any shape is a triangle if it has exactly three straight sides and is closed.

Algebra: Children recognize and duplicate simple sequential patterns (e.g., square, circle, square, circle, square, circle,…).

Curriculum Focal Points and Connections for Kindergarten

The set of three curriculum focal points and related connections for mathematics in kindergarten follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Kindergarten Curriculum Focal Points

Number and Operations: Representing, comparing, and ordering whole numbers and joining and separating sets

Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.

Geometry: Describing shapes and space

Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Measurement: Ordering objects by measurable attributes

Children use measurable attributes, such as length or weight, to solve problems by comparing and ordering objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.

Connections to the Focal Points

Data Analysis: Children sort objects and use one or more attributes to solve problems. For example, they might sort solids that roll easily from those that do not. Or they might collect data and use counting to answer such questions as, "What is our favorite snack?" They re-sort objects by using new attributes (e.g., after sorting solids according to which ones roll, they might re-sort the solids according to which ones stack easily).

Geometry: Children integrate their understandings of geometry, measurement, and number. For example, they understand, discuss, and create simple navigational directions (e.g., "Walk forward 10 steps, turn right, and walk forward 5 steps").

Algebra: Children identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships.

Curriculum Focal Points and Connections for Grade 1

The set of three curriculum focal points and related connections for mathematics in grade 1 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 1 Curriculum Focal Points

Number and Operations and *Algebra:* Developing understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts

Children develop strategies for adding and subtracting whole numbers on the basis of their earlier work with small numbers. They use a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes), and number lines, to model "part-whole," "adding to," "taking away from," and "comparing" situations to develop an understanding of the meanings of addition and subtraction and strategies to solve such arithmetic problems. Children understand the connections between counting and the operations of addition and subtraction (e.g., adding two is the same as "counting on" two). They use properties of addition (commutativity and associativity) to add whole numbers, and they create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems involving basic facts. By comparing a variety of solution strategies, children relate addition and subtraction as inverse operations.

Number and Operations: Developing an understanding of whole number relationships, including grouping in tens and ones

Children compare and order whole numbers (at least to 100) to develop an understanding of and solve problems involving the relative sizes of these numbers. They think of whole numbers between 10 and 100 in terms of groups of tens and ones (especially recognizing the numbers 11 to 19 as 1 group of ten and particular numbers of ones). They understand the sequential order of the counting numbers and their relative magnitudes and represent numbers on a number line.

Geometry: Composing and decomposing geometric shapes

Children compose and decompose plane and solid figures (e.g., by putting two congruent isosceles triangles together to make a rhombus), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine figures, they recognize them from different perspectives and orientations, describe their geometric attributes and properties, and determine how they are alike and different, in the process developing a background for measurement and initial understandings of such properties as congruence and symmetry.

Connections to the Focal Points

Number and Operations and ***Algebra:*** Children use mathematical reasoning, including ideas such as commutativity and associativity and beginning ideas of tens and ones, to solve two-digit addition and subtraction problems with strategies that they understand and can explain. They solve both routine and nonroutine problems.

Measurement and ***Data Analysis:*** Children strengthen their sense of number by solving problems involving measurements and data. Measuring by laying multiple copies of a unit end to end and then counting the units by using groups of tens and ones supports children's understanding of number lines and number relationships. Representing measurements and discrete data in picture and bar graphs involves counting and comparisons that provide another meaningful connection to number relationships.

Algebra: Through identifying, describing, and applying number patterns and properties in developing strategies for basic facts, children learn about other properties of numbers and operations, such as odd and even (e.g., "Even numbers of objects can be paired, with none left over"), and 0 as the identity element for addition.

Curriculum Focal Points and Connections for Grade 2

The set of three curriculum focal points and related connections for mathematics in grade 2 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 2 Curriculum Focal Points	Connections to the Focal Points
Number and Operations: **Developing an understanding of the base-ten numeration system and place-value concepts** Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).	***Number and Operations:*** Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition. ***Geometry*** and ***Measurement:*** Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions. ***Algebra:*** Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.
Number and Operations **and** ***Algebra:*** **Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction** Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.	
Measurement: **Developing an understanding of linear measurement and facility in measuring lengths** Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).	

Curriculum Focal Points and Connections for Grade 3

The set of three curriculum focal points and related connections for mathematics in grade 3 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

Grade 3 Curriculum Focal Points

Number and Operations and *Algebra:* Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts

Students understand the meanings of multiplication and division of whole numbers through the use of representations (e.g., equal-sized groups, arrays, area models, and equal "jumps" on number lines for multiplication, and successive subtraction, partitioning, and sharing for division). They use properties of addition and multiplication (e.g., commutativity, associativity, and the distributive property) to multiply whole numbers and apply increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving basic facts. By comparing a variety of solution strategies, students relate multiplication and division as inverse operations.

Number and Operations: Developing an understanding of fractions and fraction equivalence

Students develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line. They understand that the size of a fractional part is relative to the size of the whole, and they use fractions to represent numbers that are equal to, less than, or greater than 1. They solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or common numerators or denominators. They understand and use models, including the number line, to identify equivalent fractions.

Geometry: Describing and analyzing properties of two-dimensional shapes

Students describe, analyze, compare, and classify two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes. Students investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons. Through building, drawing, and analyzing two-dimensional shapes, students understand attributes and properties of two-dimensional space and the use of those attributes and properties in solving problems, including applications involving congruence and symmetry.

Connections to the Focal Points

Algebra: Understanding properties of multiplication and the relationship between multiplication and division is a part of algebra readiness that develops at grade 3. The creation and analysis of patterns and relationships involving multiplication and division should occur at this grade level. Students build a foundation for later understanding of functional relationships by describing relationships in context with such statements as, "The number of legs is 4 times the number of chairs."

Measurement: Students in grade 3 strengthen their understanding of fractions as they confront problems in linear measurement that call for more precision than the whole unit allowed them in their work in grade 2. They develop their facility in measuring with fractional parts of linear units. Students develop measurement concepts and skills through experiences in analyzing attributes and properties of two-dimensional objects. They form an understanding of perimeter as a measurable attribute and select appropriate units, strategies, and tools to solve problems involving perimeter.

Data Analysis: Addition, subtraction, multiplication, and division of whole numbers come into play as students construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to solve problems.

Number and Operations: Building on their work in grade 2, students extend their understanding of place value to numbers up to 10,000 in various contexts. Students also apply this understanding to the task of representing numbers in different equivalent forms (e.g., expanded notation). They develop their understanding of numbers by building their facility with mental computation (addition and subtraction in special cases, such as 2,500 + 6,000 and 9,000 – 5,000), by using computational estimation, and by performing paper-and-pencil computations.

Counting and Cardinality

Kindergarten

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
    a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
    c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

[1] Include groups with up to ten objects

Number and Operations in Base Ten

Kindergarten

Work with numbers 11–19 to gain foundations for place value.

1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Grade 1

Extend the counting sequence.

1 Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

- a 10 can be thought of as a bundle of ten ones—called a "ten."
- b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
- c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.

Use place value understanding and properties of operations to add and subtract.

4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

6 Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Grade 2

Understand place value.

1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

- a 100 can be thought of as a bundle of ten tens—called a "hundred."
- b The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2 Count within 1000; skip-count by 5s, 10s, and 100s.

3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9 Explain why addition and subtraction strategies work, using place value and the properties of operations.[1]

Grade 3

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1 Use place value understanding to round whole numbers to the nearest 10 or 100.

2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

Grade 4

Generalize place value understanding for multi-digit whole numbers.

1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that* $700 \div 70 = 10$ *by applying concepts of place value and division.*

2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

3 Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

[1] Explanations may be supported by drawings or objects.

6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Grade 5

Understand the place value system.

1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

3 Read, write, and compare decimals to thousandths.

   a Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

   b Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

4 Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5 Fluently multiply multi-digit whole numbers using the standard algorithm.

6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Geometry

Kindergarten

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.

2 Correctly name shapes regardless of their orientations or overall size.

3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

6 Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Grade 1

Reason with shapes and their attributes.

1 Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size) ; build and draw shapes to possess defining attributes.

2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[1]

3 Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Grade 2

Reason with shapes and their attributes.

1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[2] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[1] Students do not need to learn formal names such as "right rectangular prism."

[2] Sizes are compared directly or visually, not compared by measuring.

Grade 3

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Grade 4

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade 5

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g.,x-axis and x-coordinate, y-axis and y-coordinate).

2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

4. Classify two-dimensional figures in a hierarchy based on properties.

Grade 6

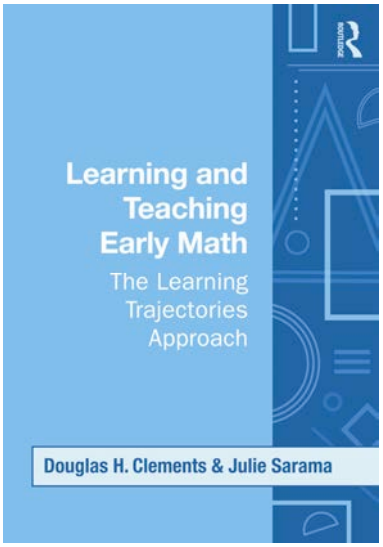
Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

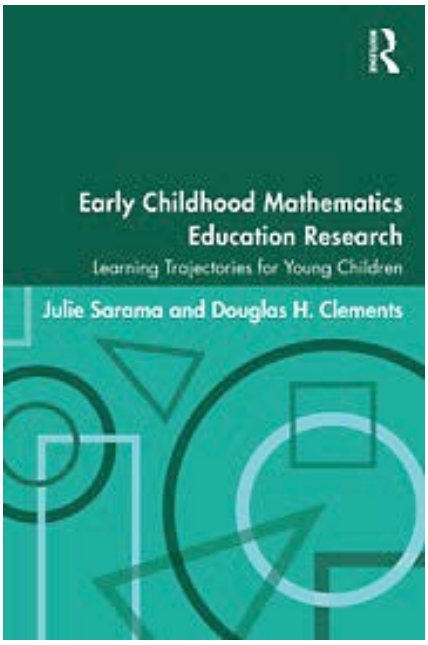
Arithmetic Games from:

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach.* New York: Routledge.

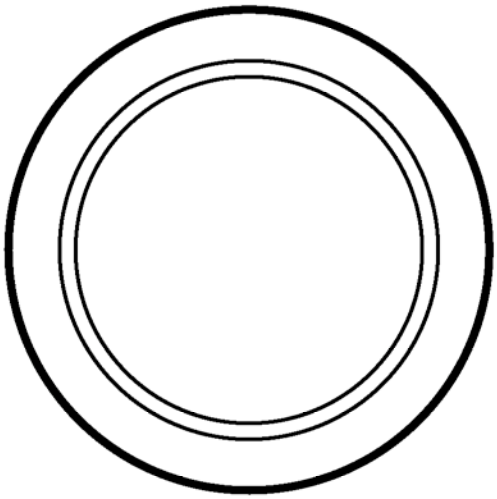


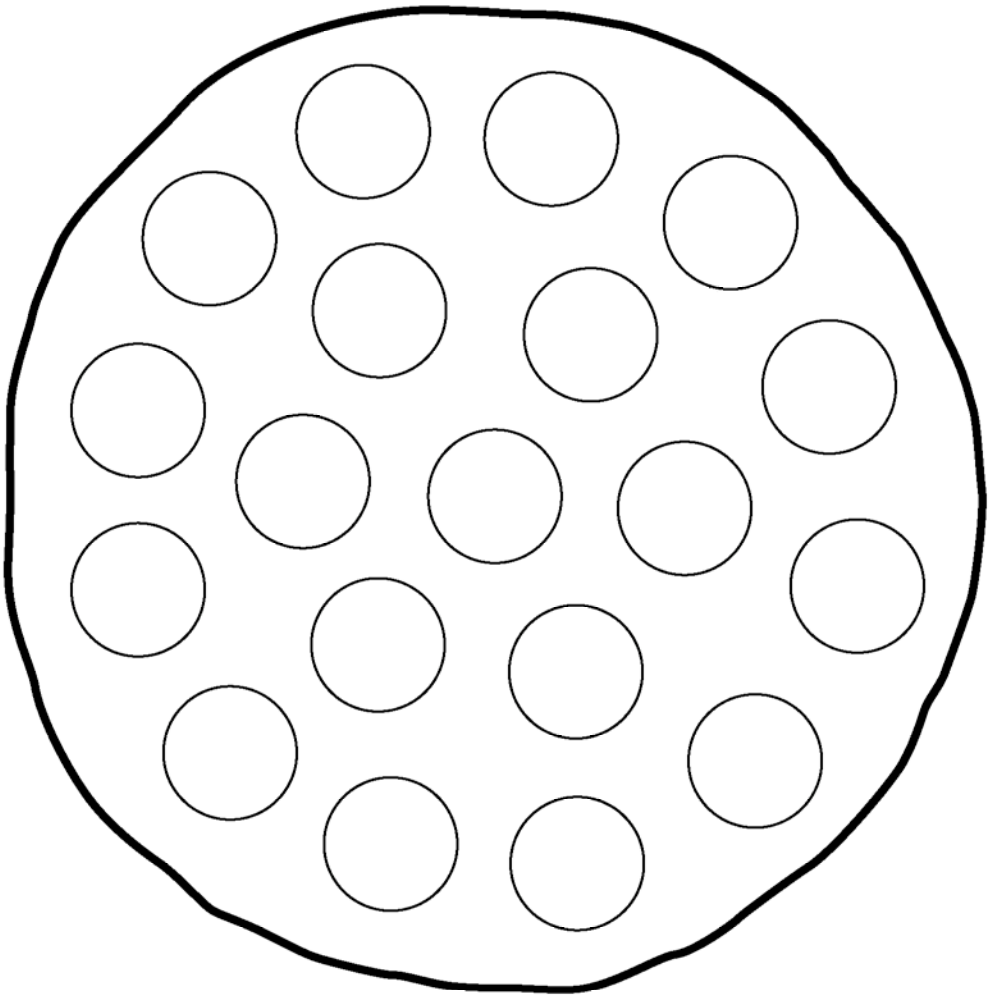
See also:

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children.* New York: Routledge.

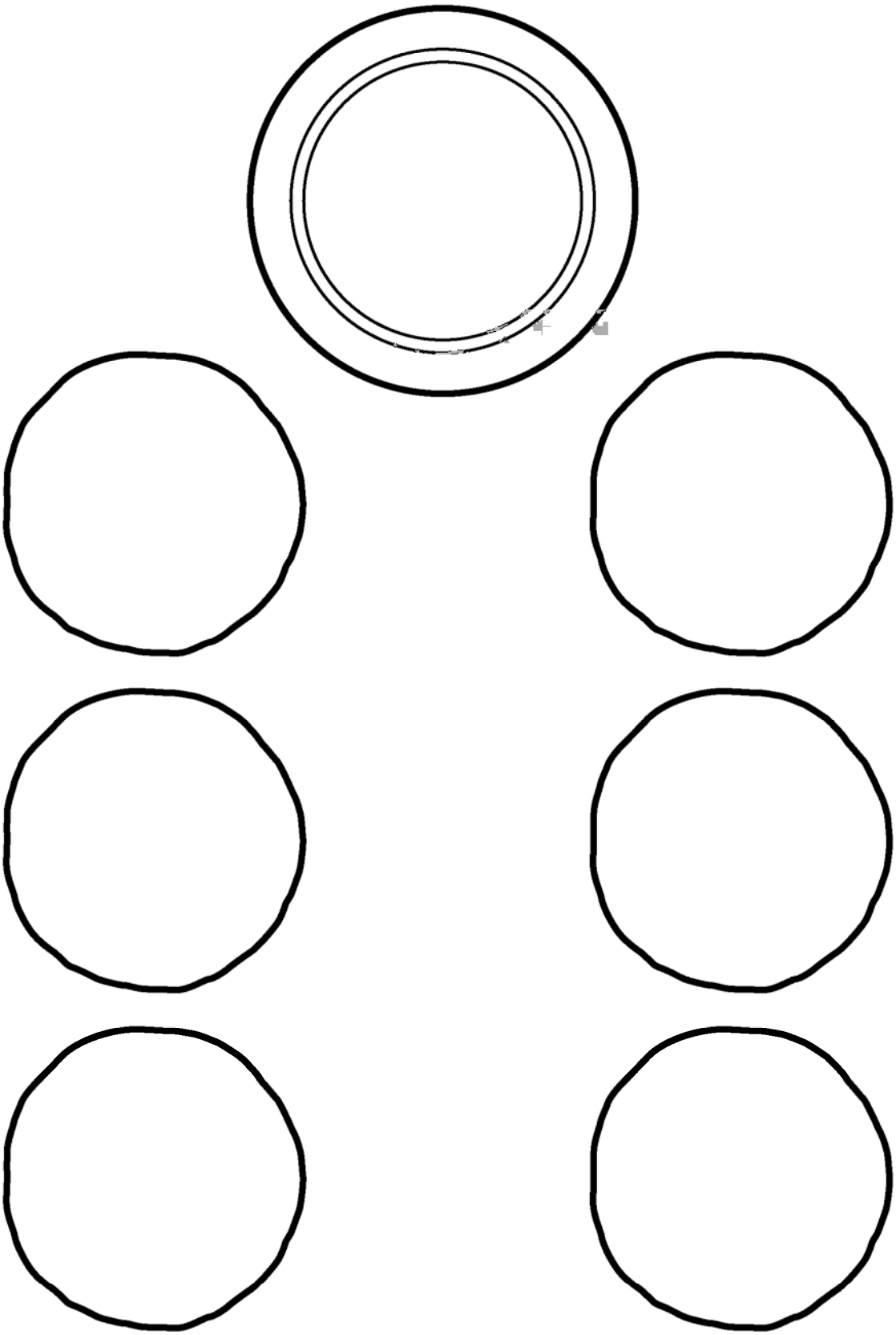


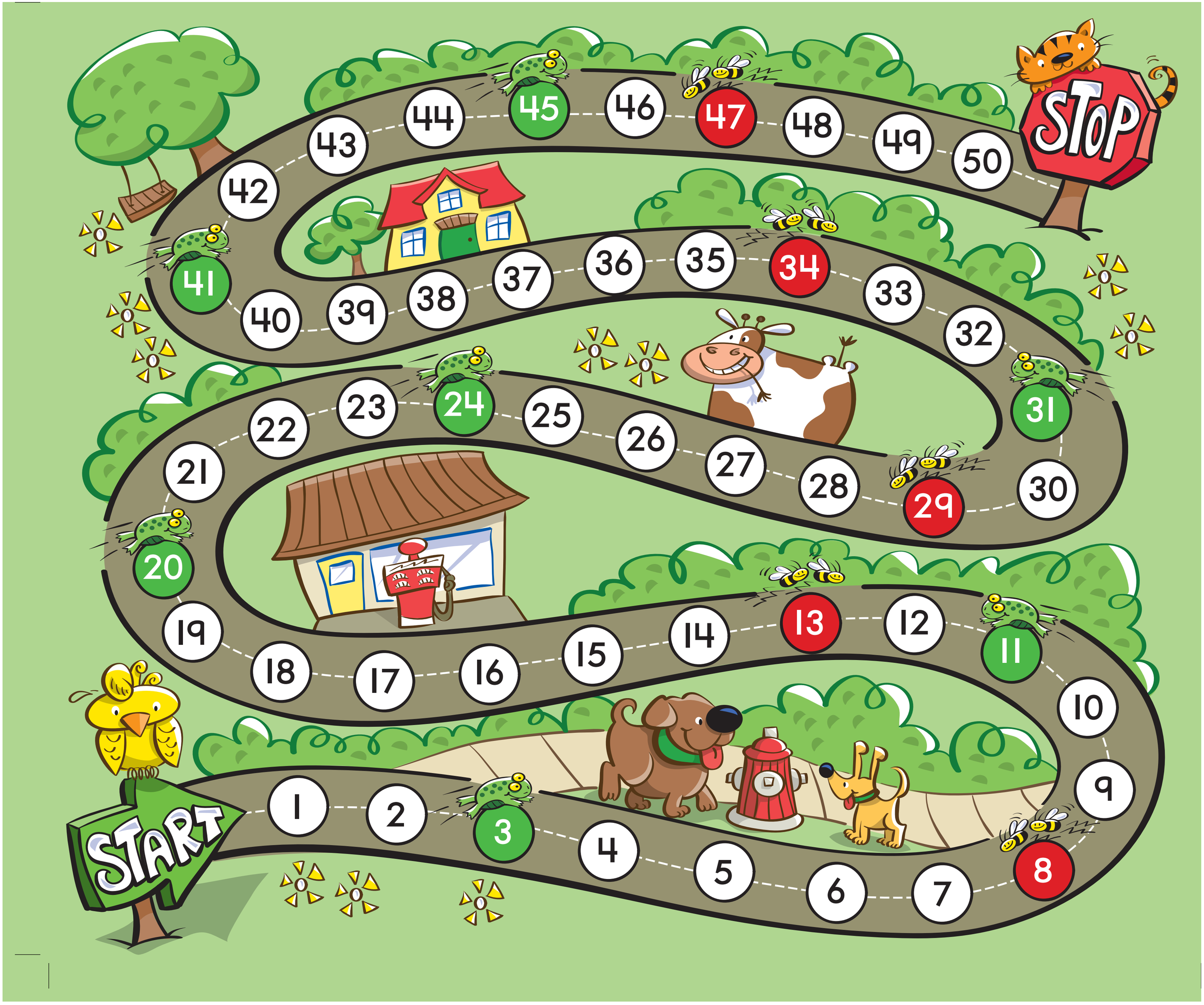
Pizza Game 1





Pizza Game 2

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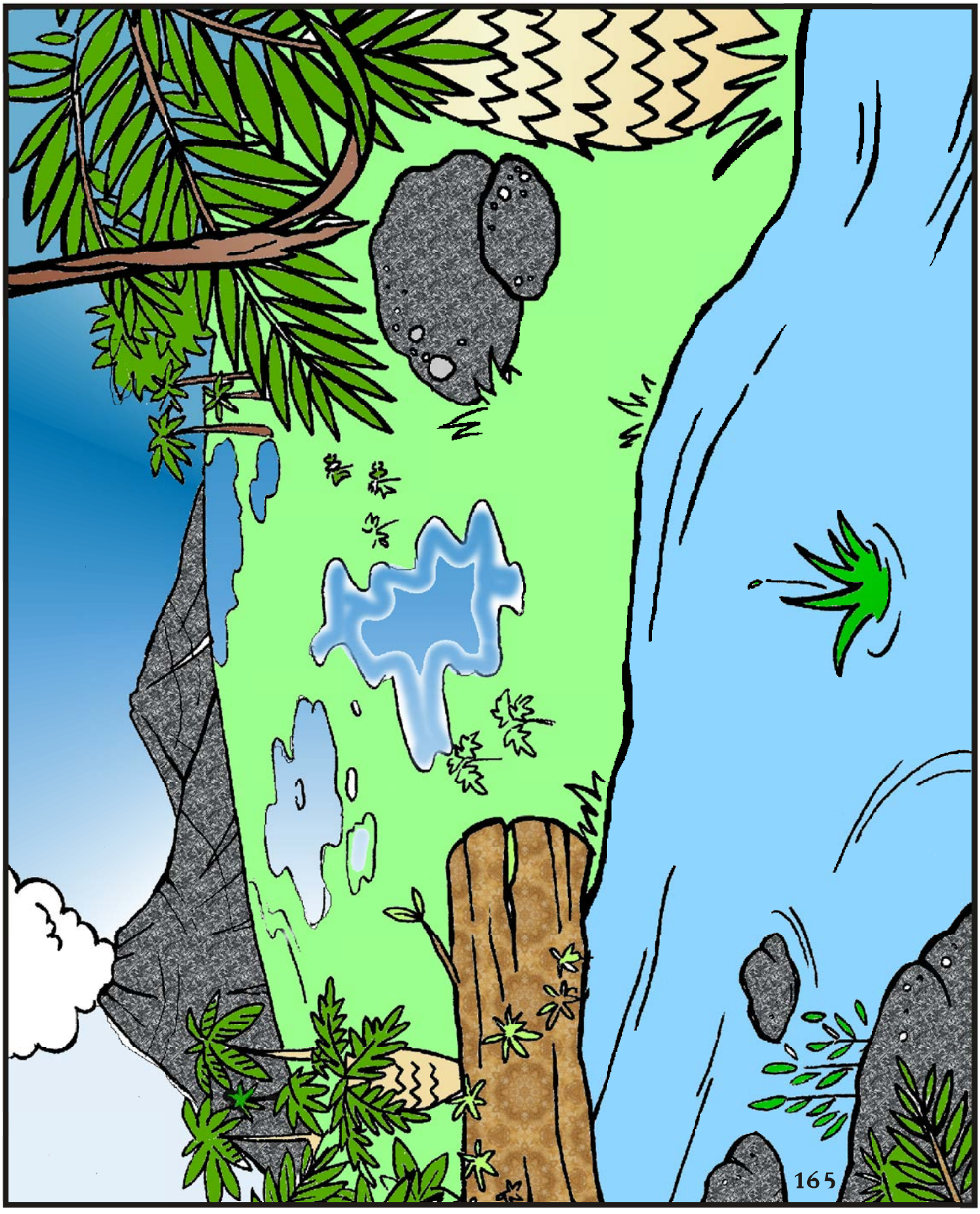
STOP

Places Scene: Dinosaur



Slap a 10

Materials: Deck of Number Cards (4 each of 1-10) (remove the Wild Cards)

Players: 2-4

Object: Combinations (missing addend)

Note to families:
In this game, your child will be finding the totals of pairs of numbers. You will need a set of Number Cards to play this game.

How to Play

Introduce this 2- to 4-player game.

Six cards are dealt out to each player. The remaining cards are placed in the middle, face down.

One player turns the top card over. The other players quickly determine if they can make a ten with that and one card in their hand. If they can, they slap the card. The player who slaps it first must use it to make a ten. If they cannot, they keep the card and must take another card off the pile.

Players take turns turning over the top card.

The game ends when player goes "out" or the pile is gone. The player who went "out" or the one with the fewest cards in their hand wins.

Modification: If children are having a problem trying to slap the card at the same time…

If they can make a ten with the card shown, they slap their own card down. The player who slapped it down first will ask "Is it 10?"

All players must agree that the two cards make 10.

Fish for 10

Like "Go Fish" but with sums of 10.

Tic-Tac-Total

Draw a tic-tac-toe board and write the numbers 1 to 10. Players take turn crossing out one of the numbers and writing it in the board. Whoever makes 15 first wins.

Salute!

Materials:	Deck of Number Cards (4 each of 0-10) (remove the Wild Cards)	**Note to families:** In this game, your child will be finding the totals of pairs of numbers. You will need a set of Number Cards to play this game.
Players:	3	
Object:	Combinations (missing addend)	

How to Play

1. Mix the cards and deal them evenly to 2 of the 3 players. They place their stacks of cards face down in front of them
2. The two players sit facing each other with their cards face down.
3. The third player says, “Salute!” and the two players take the top card from their piles and hold them on their foreheads so that the other two players can see them, but they cannot.
4. The third player announces the sum of the two cards. At this point, the race is on: Each of the other players tries to be the first to announce the value of their own cards. The person who is first takes both cards.
5. The winner is the person who collects the most cards.

Gentle introduction: Play first where each player can get a point by figuring out their own card (no “race”).

Double Compare Instructions

Materials:	Deck of Number Cards (4 each of 0-10) (remove the Wild Cards)	**Note to families:** In this game, your child will be finding the totals of pairs of numbers. You will need a set of Number Cards to play this game.
Players:	2	
Object:	Decide which of two sums is greater.	

How to Play

1. Mix the cards and deal them evenly to each player. Place your stack of cards face down in front of you.

2. At the same time, both of you turn over the top two cards in your stack. Compare your cards to your partners to determine which sum is more. If your total is more than the other player's, say "Me!" If the two totals are the same, turn over the next two cards and compare these sums.

   Sometimes you may be able to decide which pair is more without actually figuring out the total.

3. Keep turning over two cards. Say "Me!" each time your total is more.

4. The game is over when you have both turned over all the cards in your stack.

Variations

- Remove the 7-10 cards from the deck, and play with just the 0-6 cards.

- Play Compare. Players turn over one card on a turn. The player with the larger number says "Me!"

- Add the four wild cards to the deck. A wild card may be used as any number. Challenge students to use it for the lowest number that will allow them to win.

- Play Triple Compare. Players turn over three cards on a turn. The player with the larger total says "Me!."

Four in a Row

Materials: Four in a Row Sheet; chips in 2 colors
Players: 2

How to Play

Each player has chips of one color ("see through" if possible).

Each chooses two numerals in the square on the left, summing them and covering them with chips. The player also covers the sum on the square on the right.

The first to make four in a row is the winner

5	6	7
8	9	10
11	12	13

16	21	18	13	18
19	20	12	20	23
22	24	19	21	16
17	11	23	22	14
14	15	15	17	25

Counting Cards

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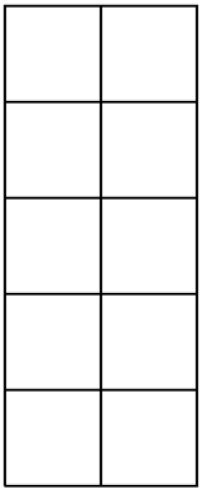

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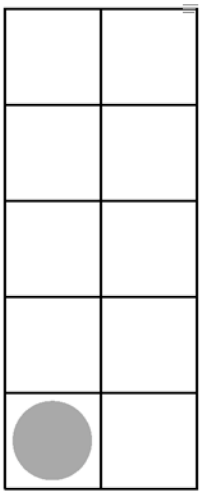

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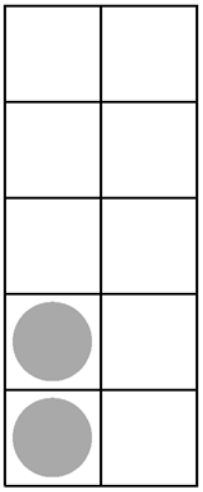

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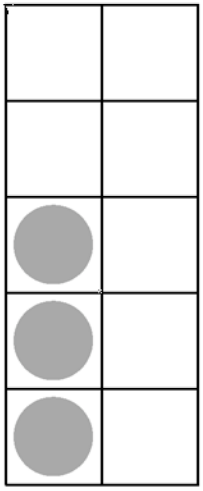

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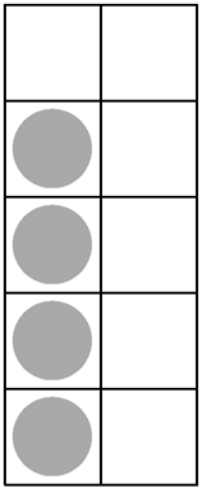

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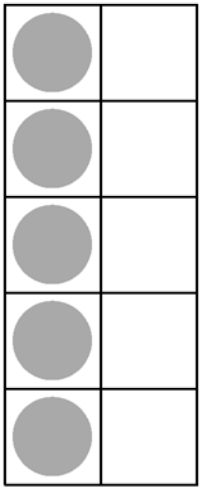

Counting Cards

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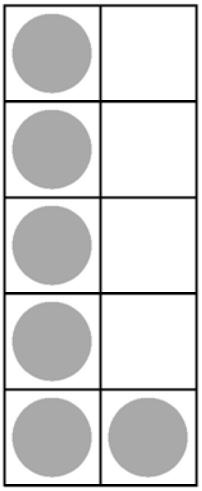

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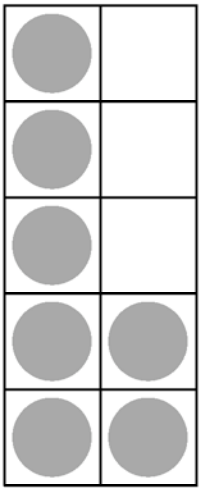

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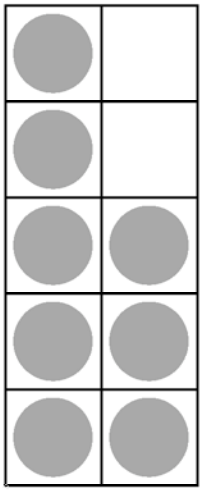

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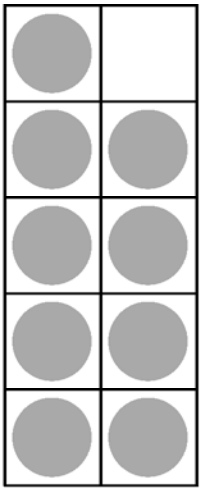

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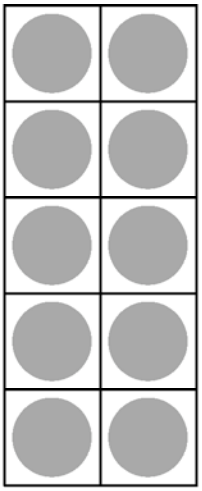

Close to 100

Materials: One deck of Numeral Cards
Close to 100 Score Sheet for each player
Players: 1, 2, or 3

How to Play

1. Deal out six numeral cards to each player.
2. Use any four cards to make two numbers. For example, a 6 and a 5 could make 65 or 56. Wild cards can be used as any numeral. Try to make numbers that, when added, give you a total that is close to 100.
3. Write these numbers and their total on the Close to 100 Score Sheet. For example: 42 + 56 = 98.
4. Find your score. Your score is the difference between your total and 100.
5. Put the cards you used in a discard pile. Keep the two cards you didn't use for the next round.
6. For the next round, deal four new cards to each player. Make more numbers that come close to 100. When you run out of cards, mix up the discard pile and use them again.
7. After five rounds, total your scores. The lowest score wins.

Variations

Early Childhood Version

Play close to 10, dealing out 4 cards and using the two with a sum closest to 10. Find the difference, take that many counters. The fewest counters wins.

Early 2nd Grade

Play close to 20, dealing out 5 cards and using the three with a sum closest to 20.

Alternate Scoring

Write the score with plus and minus signs to show the direction of your total away from 100. For example: If your total is 98, your score is -2. If your total is 105, your score is +5. The total of these two scores would be +3. Your goal is to get a total score for five rounds that is close to 0.

Close to 1000

Make two numbers that total as close as possible to 1000. For example, if you are dealt 4, 5, 8, 3, 2, 9, 9, and 0, you might try 420 + 583 (1003). The score for each round is the difference of the total and 1000. In the example above, the score is 3. Play five rounds. The total lowest score wins. Again, as students become comfortable with this version, negative and positive integers may be introduced.

Close to 0 with Two or Three Digit Numbers

To play with 2-digit numbers, deal out 6 cards to each player. Each player uses any four cards to make two numbers whose difference is as close as possible to 0. For 3-digit numbers, deal out 8 cards to each player. Each player uses any six cards to make two numbers whose difference is as close as possible to 0. With the numbers above, a player might try 402 - 399. Three is 3 away from 0, so the player's scored would be 3. Play five rounds and the lowest total score wins.

Other K-2 Number Games

Games can support students in developing and practicing important concepts and skills in number, geometry, and data. The games described here for grades K-2 all use the Primary set of number cards that contain numbers and pictures. Some of the games include variations that you can use to adjust the level of challenge; you might decide to create your own variations. As you observe your students playing, think about what students are learning and ways that they are demonstrating their mathematical understandings.

- *Card Tricks*. From *Building Blocks*. A series of simple number card activities.
  Ordering cards. Give each child a collection of cards 1-5. They put them in order face up. Alternatively, have them work in pairs, with each partner putting down the next card in turn. Work up to all cards 1-10.
  Magically knowing the missing card. With their cards in numerical order, children take turns hiding a card when their partner's eyes are closed. The partner identifies which card disappeared. At first, the space is left open; later, have children space the remaining cards equally.
  X-ray vision. Children sit in pairs, order the cards and turn them face down. Child one points to any of the other's cards, whereupon child two names it. That card is turned over, and the children switch roles.
  Feel the Number. One child chooses two cards and figures out their sum, which she/he writes down. She/he places one face up and one face down. She/he tells the partner the sum and the partner must identify the face-down card. Children then switch roles.
  Card Game. Practice these skills with a game. Use four collections of cards (0 to 5 or 0 to 10, depending on the children's level). One collection is placed face up in numerical order. The remaining cards are distributed among the players. Each player in turn can put down any one card they have which is the "next smallest number" for that row. If a player has no appropriate card, he or she loses that turn. The first one "out" is the winner.

- *Go Fish for Ten* From *Building Blocks*. The object is to find pairs of cards that total 10 (such as, 6 and 4, or 7 and 3). After ten cards are dealt to each player, the remaining cards are placed face down in the fishpond. Players find all pairs of ten in their hands and place each pair in their own score pile. Player 1 asks player 2 for a card needed to form a pair of 10. If player 2 has the card asked for, s/he must give it to player 1. If not, player 2 responds "Go fish" and player 1 draws a card from the fish pond. Either way, if player 1 obtains the card needed, he puts the cards in his/her "score pile" and his/her turn continues. If the desired card is not obtained, it is now player 2's turn. Play continues until a player has formed pairs of 10 using all of the cards in his/her hand. The player with the most pairs is the winner.

- *Total of 10* A deck of Number Cards is laid out face up in rows of 5. Students take turns finding combinations of cards that total 10 (such as, 5, 2, and 3, or 6, 2, 1, and 1). At the end they write down all the combinations they made using addition notation. This game is included in Grade 1 *(Number Games and Story Problems),* and a very similar game, Turn Over 10, played like Concentration, is include in Grade 2 *(Mathematical Thinking at Grade 2,* and *Coins, Coupons, and Combinations). From Investigations in Number, Data, and Space*

- *Close to 20* (Grade 1: *Number Games and Story Problems* (extension), Grade 2: *Coins, Coupons, and Combinations*, page 143 and 144) This game is a variation of Close to 100. It uses the K-2 number cards and interlocking cubes. From 5 cards, players each chooses three cards that total as close to 20 as possible. Their score for the round is the difference between the total and 20, and they take that number of cubes. The winner is the player with the fewest cubes (the lowest score) after five rounds. *From Investigations in Number, Data, and Space*

- *Collect 25* Together (Grade 1: *Number Games and Story Problems*, page 64; and Grade 2: *Coins, Coupons, and Combinations*, page 80) In this game, pairs take turns throwing 2 dice or turning over a number card and collecting coins with that amount in cents. The goal is to cooperatively collect 25 cents or just over. For more challenge, students collect a larger amount such as 50 cents. From *From Investigations in Number, Data, and Space*

Four in a Row

5	6	7
8	9	10
11	12	13

16	21	18	13	18
19	20	12	20	23
22	24	19	21	16
17	11	23	22	14
14	15	15	17	25

Pattern Block Puzzles

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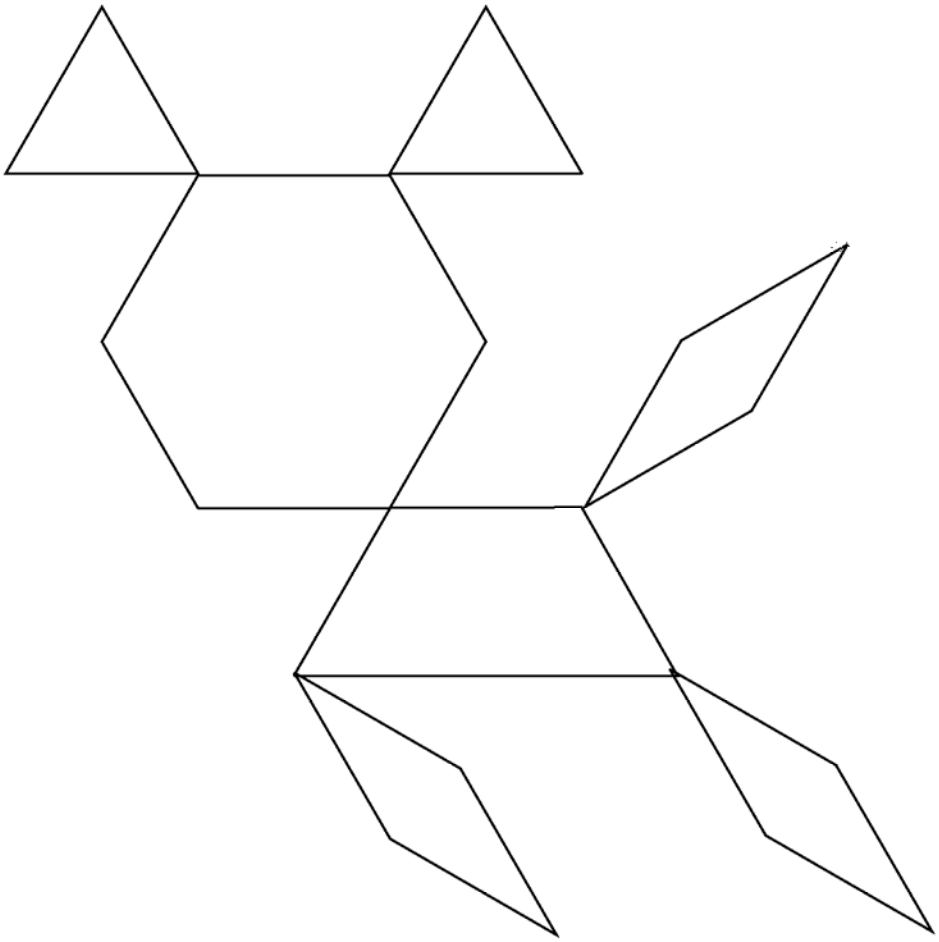

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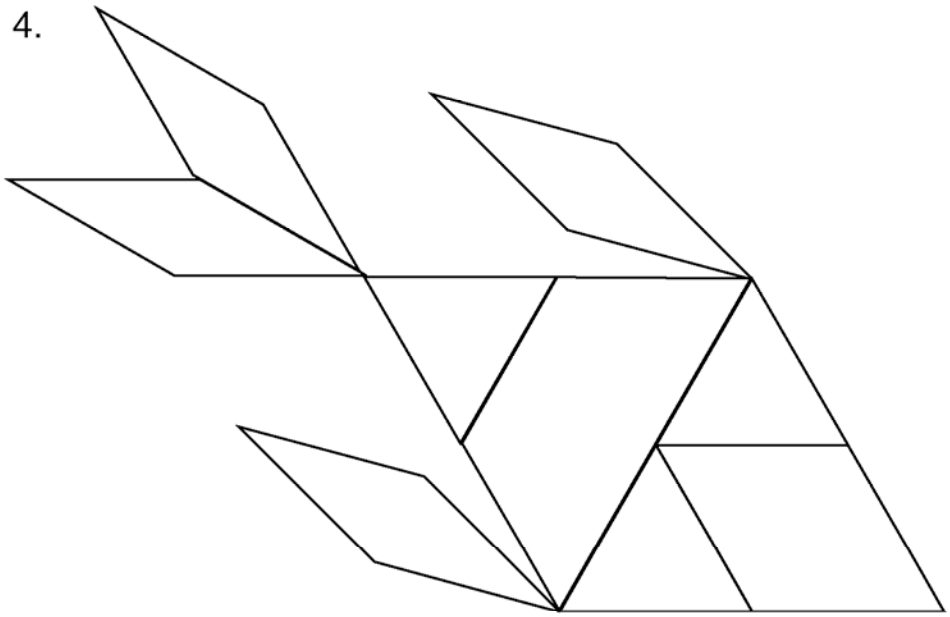

Pattern Block Puzzles

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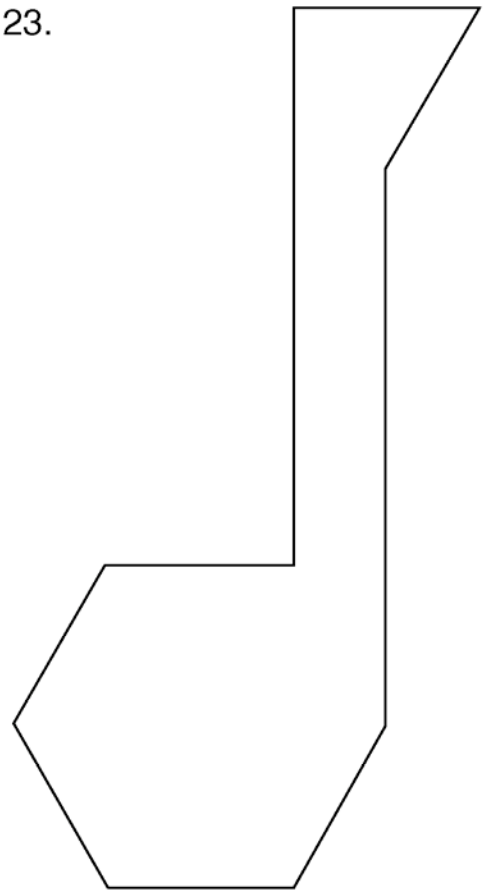

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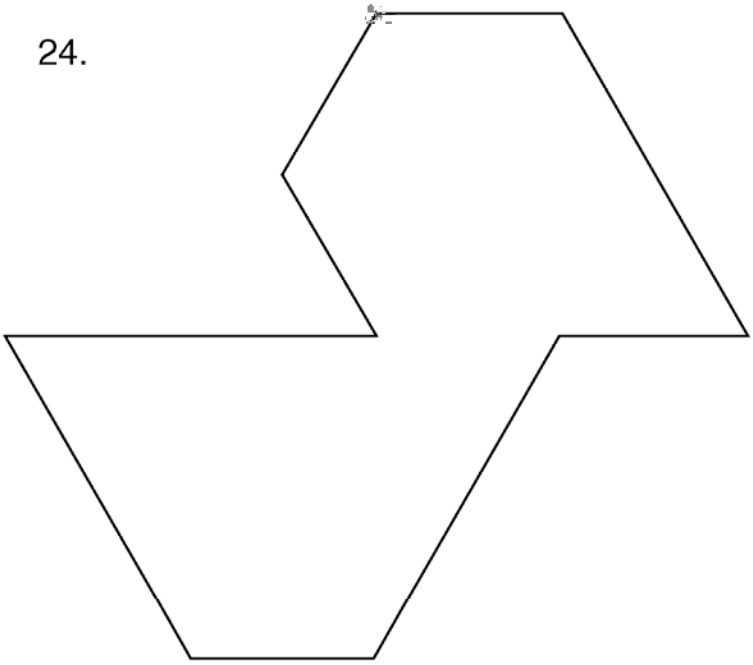

Pattern Block Puzzles

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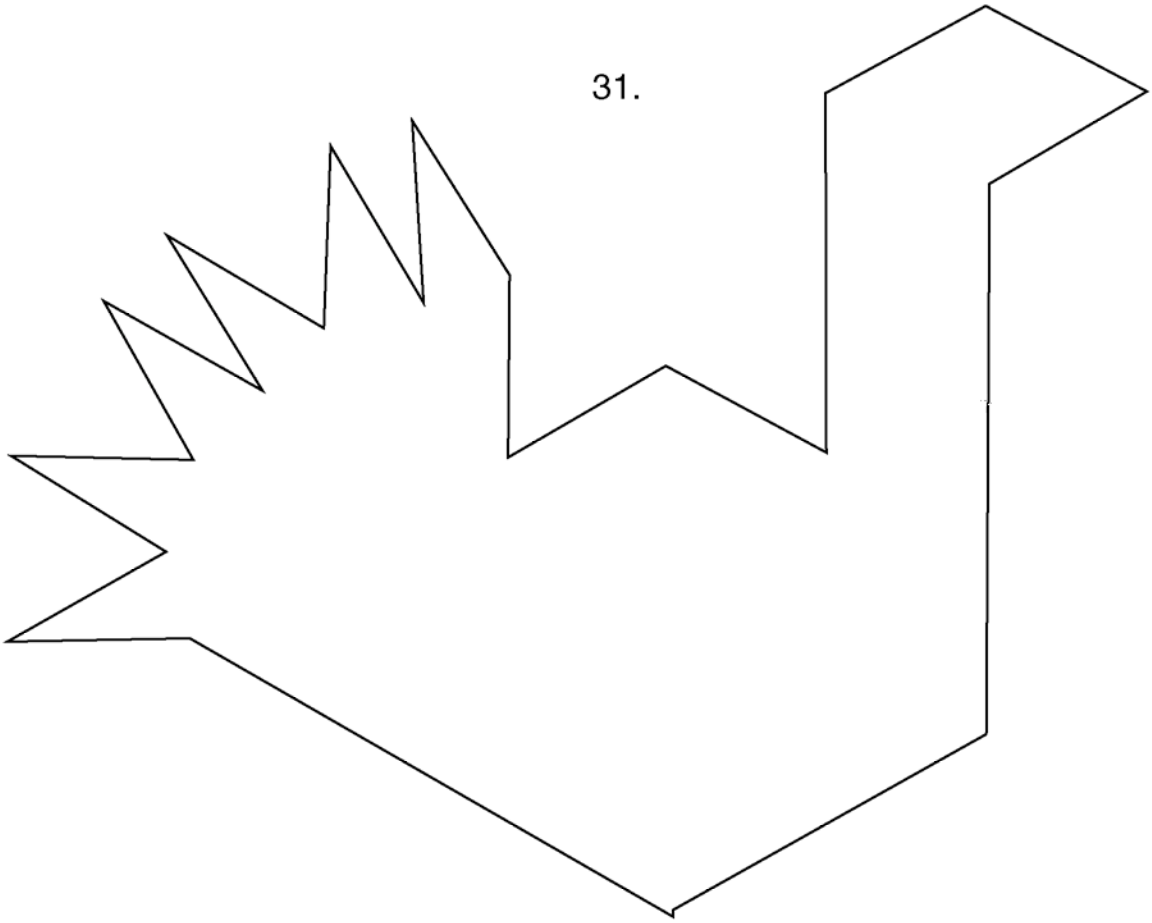

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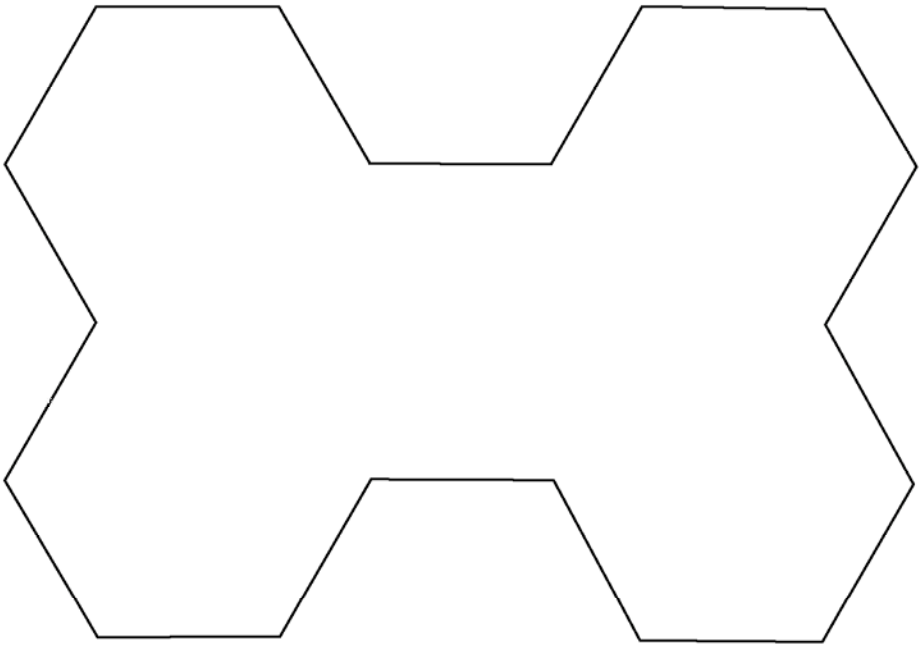
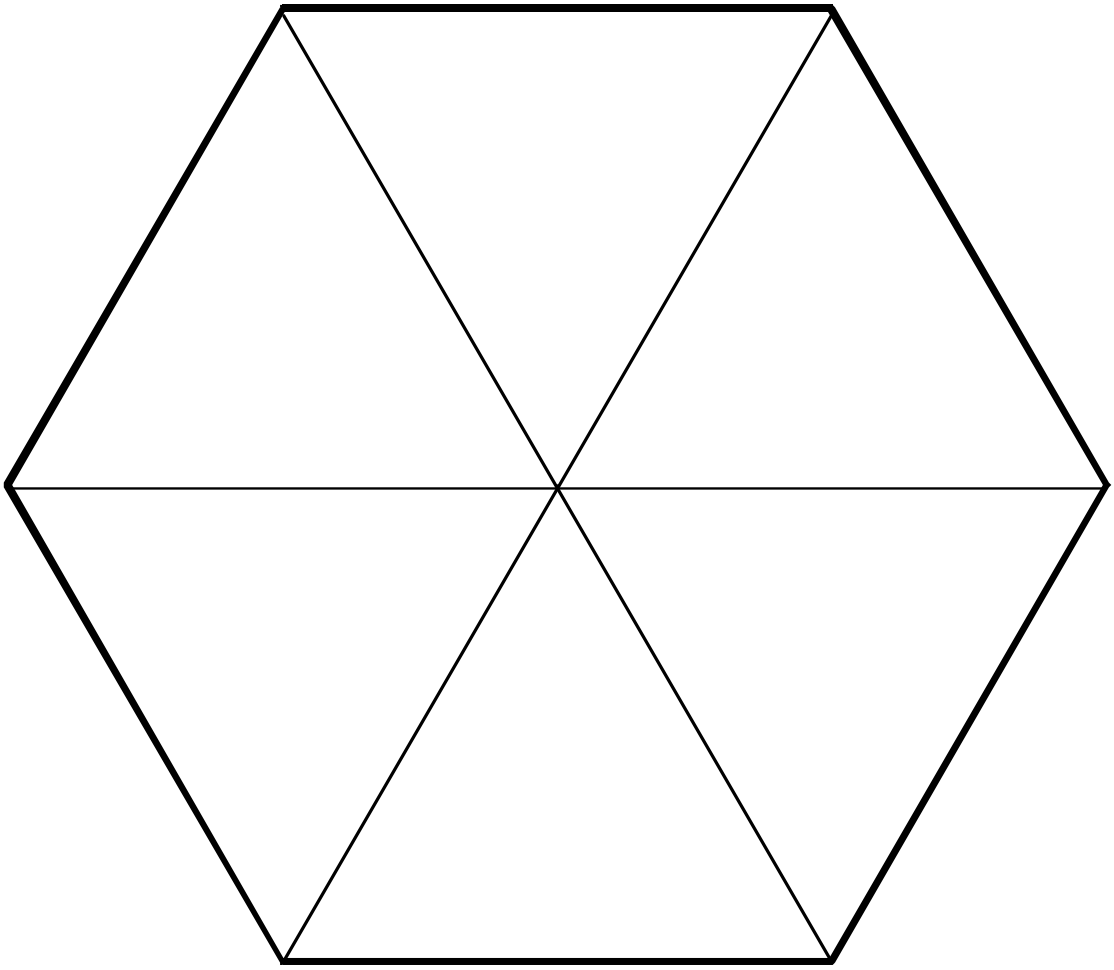


Regular hexagon, with diagonals Doug Clements



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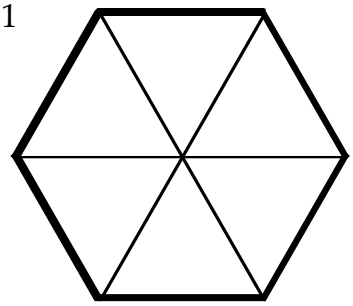

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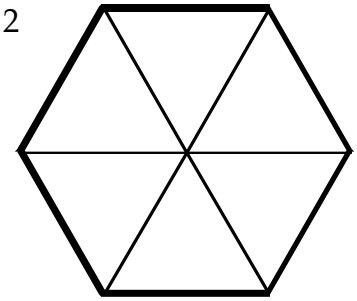

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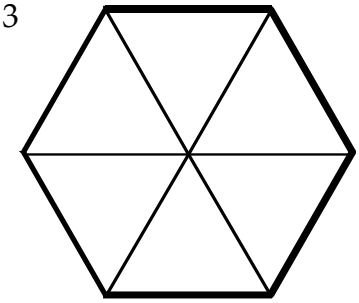

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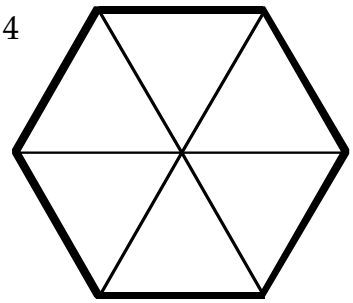

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