Math Foundations: The magic of the math

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What the Research Says

According to the 2011 National Assessment of Educational Progress (NAEP) only 42% of fourth-grade students and only 34% of eighth-grade students scored at the proficient level in mathematics. (NC: 43%;36%)

(National Center for Educational Statistics, 2009)

The National Council for Teachers of Mathematics (NCTM)

highlights the need for a well designed curriculum and quality teacher preparation.

Federal Recognition of Lack of Research Studies

- IDEIA 2004 and NCLB clearly define a high standard for research-based reading practices

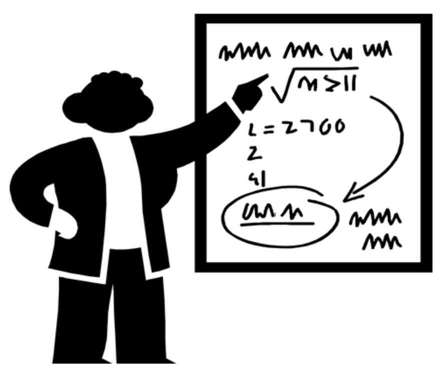
- IDEIA 2004 did not clearly define a high standard for research-based math practices, because we did not have the same research for math as we did for reading.

What Works clearinghouse:

http://www.whatworks.ed.gov

Math Basics

International Research



TIMSS

from Improving Mathematics Instruction (Ed Leadership 2/2004)

1995 Video Study
Japan, Germany, US
Teaching Style Implicated

1999 Video Study
US, Japan, Netherlands, Hong Kong, Australia, Czech Rep.
Implementation Implicated

Style vs.. Implementation

High Achieving countries use a variety of styles to teach (calculator vs.. no calculator, 'real-life' problems vs.. 'naked' problems)

High Achieving countries all implement connections problems as connections problems

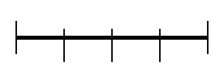
U.S. implements connection problems as a set of procedures

What is 4^2?

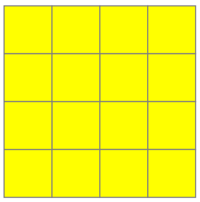
Exponents and Geometry

What is 4^2?
Why is it 4 x 4 when it looks like 4 x 2?

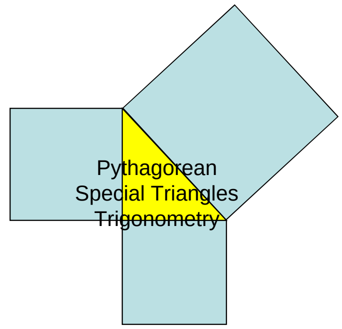
It means
'make a square out of your 4 unit linear side'



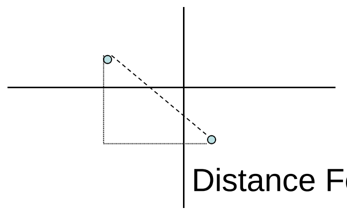
Exponents and CONNECTIONS



Square Roots! √16 = 4
The length of one side!



Geometry and
Measurement

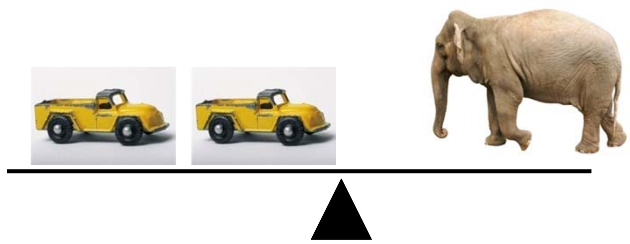


Distance Formula

Public Schools of North Carolina

$$1 = 1^3$$

Are these the same?



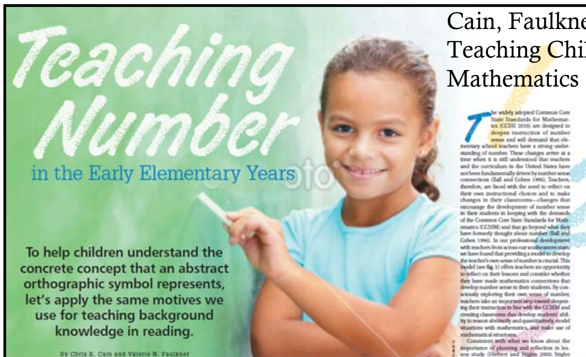
How do we fix the problems?

Teach through Number Sense

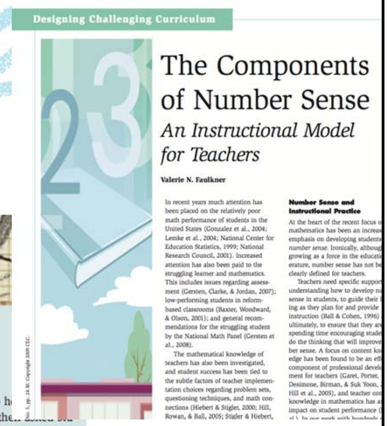
Language Connections!

Classroom Implementation of the **Components of Number Sense.**

Cain, Faulkner in Teaching Children Mathematics



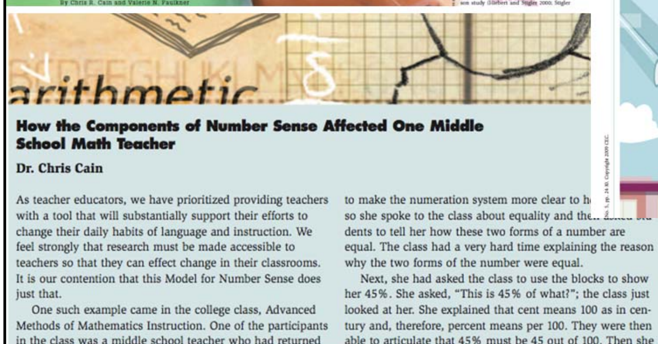
Faulkner in Teaching Exceptional Children


Designing Challenging Curriculum

The Components of Number Sense

An Instructional Model for Teachers

Valerie N. Faulkner


How the Components of Number Sense Affected One Middle School Math Teacher

Dr. Chris Cain

As teacher educators, we have prioritized providing teachers with a tool that will substantially support their efforts to change their daily habits of language and instruction. We feel strongly that research must be made accessible to teachers so that they can effect change in their classrooms. It is our contention that this Model for Number Sense does just that.

One such example came in the college class, Advanced Methods of Mathematics Instruction. One of the participants in the class was a middle school teacher who had returned

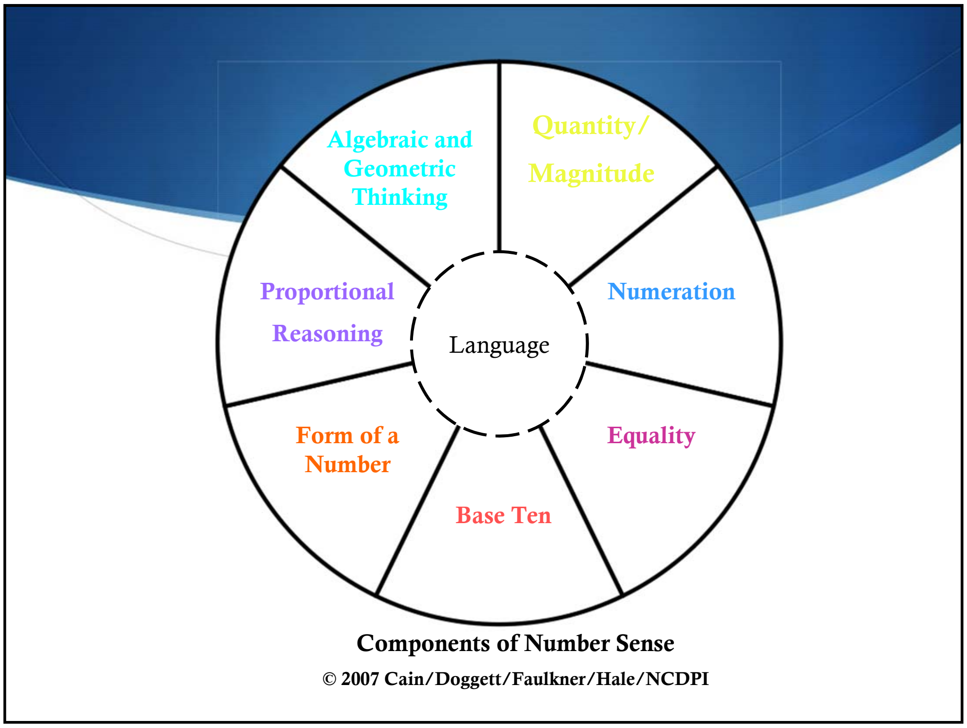
to make the numeration system more clear to h... so she spoke to the class about equality and the... dents to tell her how these two forms of a number are equal. The class had a very hard time explaining the reason why the two forms of the number were equal.

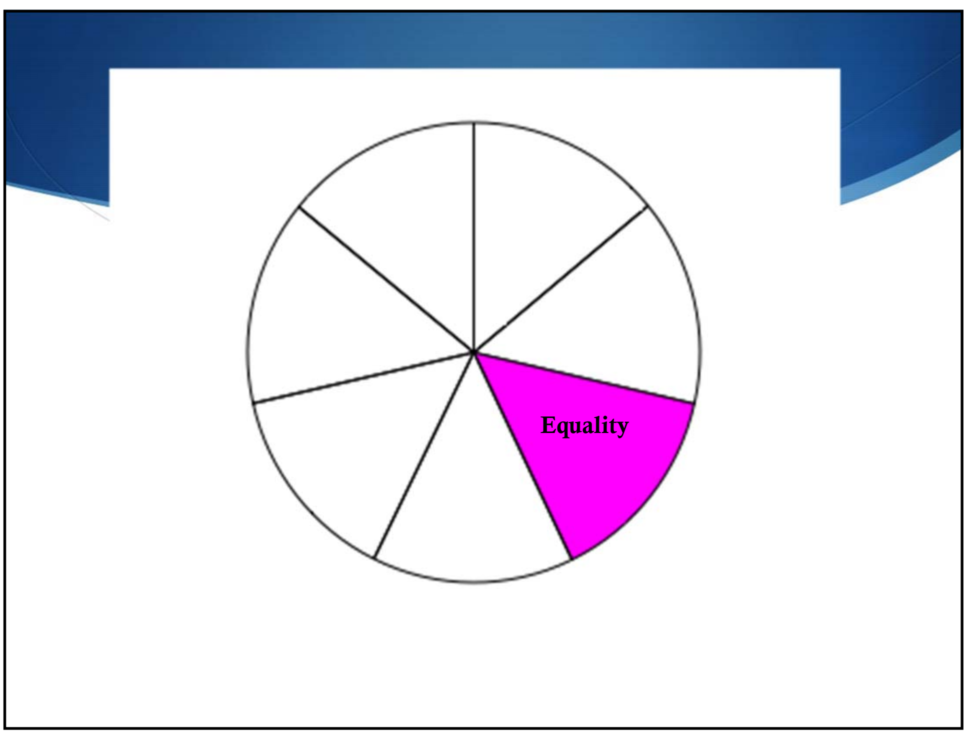
Next, she had asked the class to use the blocks to show her 45%. She asked, "This is 45% of what?"; the class just looked at her. She explained that cent means 100 as in century and, therefore, percent means per 100. They were then able to articulate that 45% must be 45 out of 100. Then she

Cain in Teaching Exceptional Children

TEDxNCSU Available on TED Website
Let's teach the substance behind the mathematics

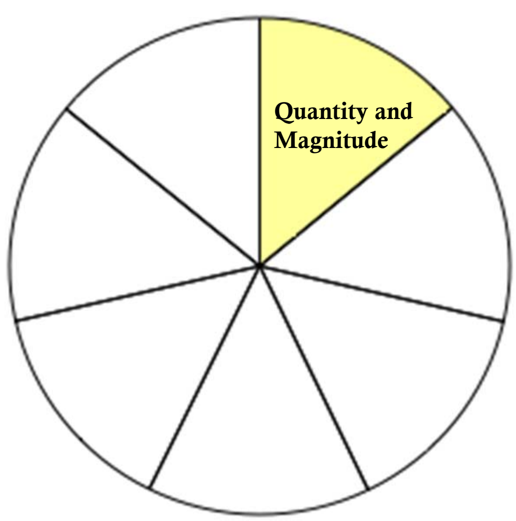
http://tedxtalks.ted.com/video/TEDxNCSU-Valerie-Faulkner-Teach


Algebraic and Geometric Thinking
Quantity/ Magnitude
Proportional Reasoning
Language
Numeration
Form of a Number
Equality
Base Ten
Components of Number Sense
© 2007 Cain/Doggett/Faulkner/Hale/NCDPI


Equality

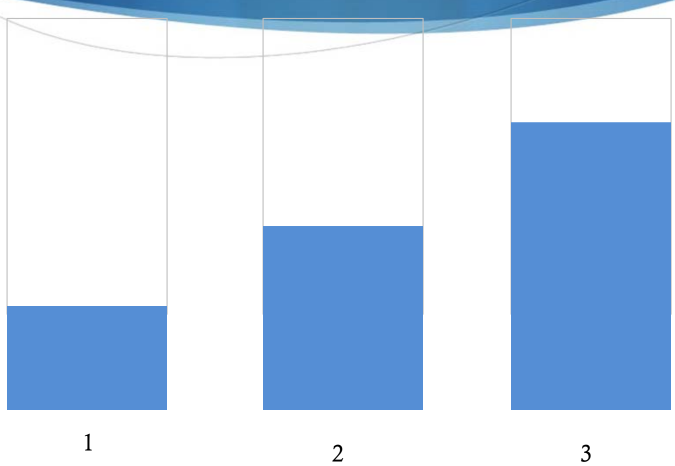
Are these the same?

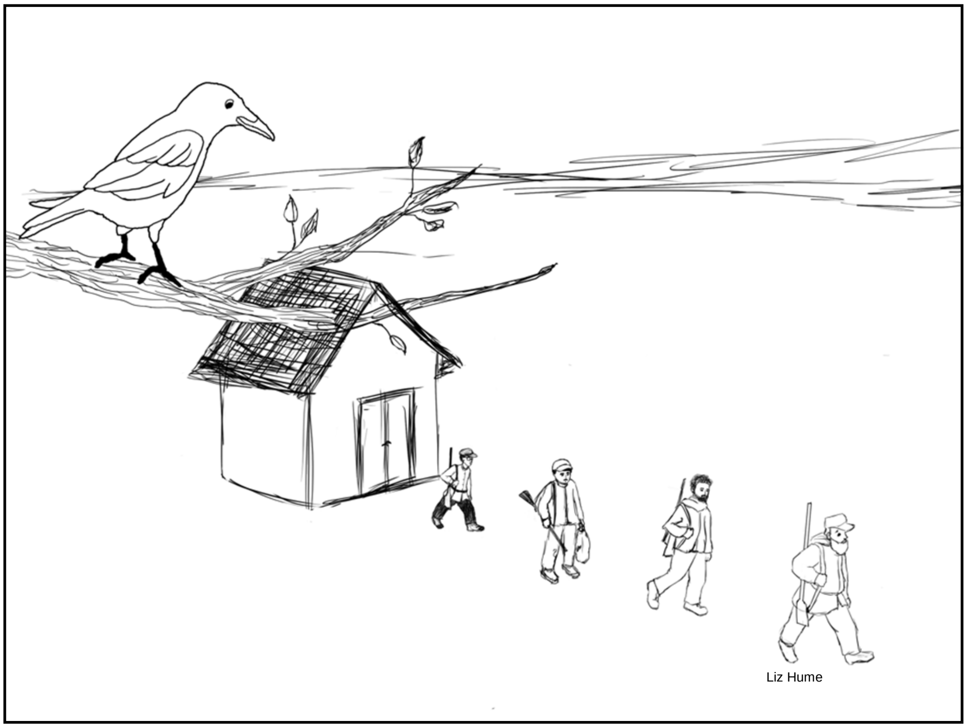
$$4+4 = 7+1$$

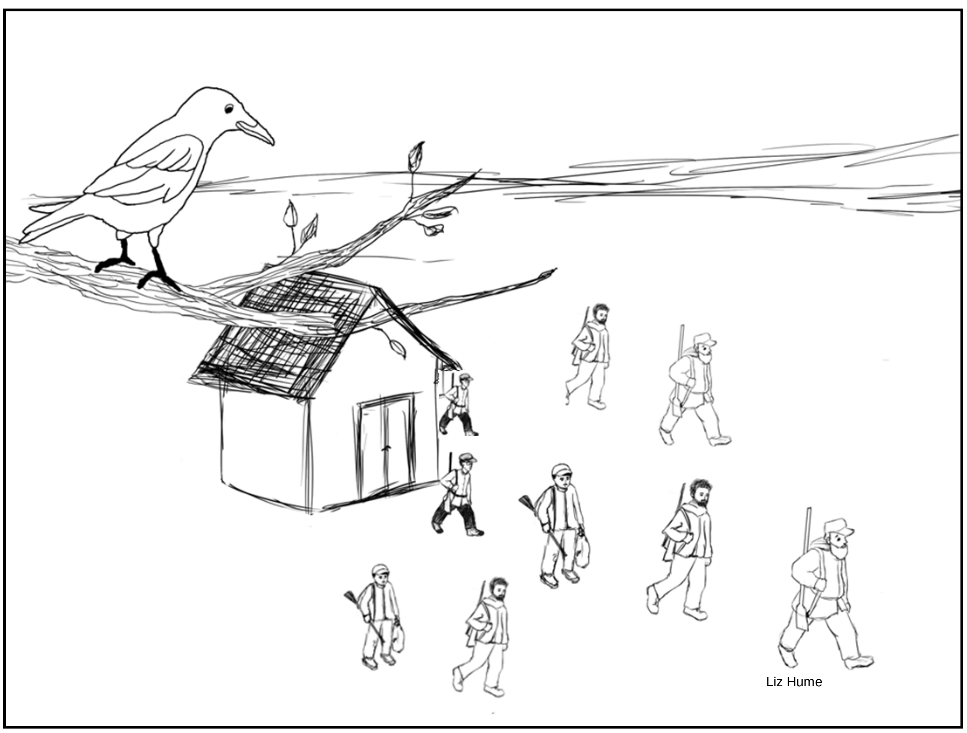


We teach *digitally* but we ALL have **analog** brains!

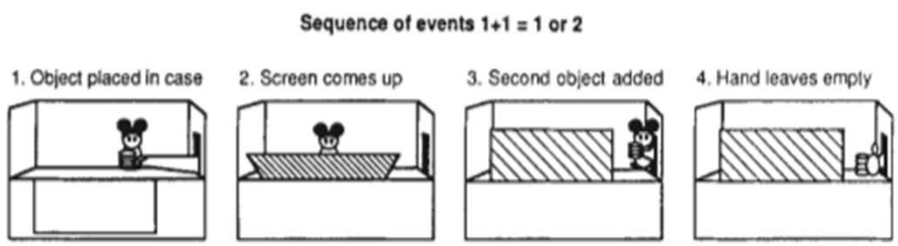
The Accumulator Model: Our Analog Brain




Liz Hume


Liz Hume

Babies can Add!

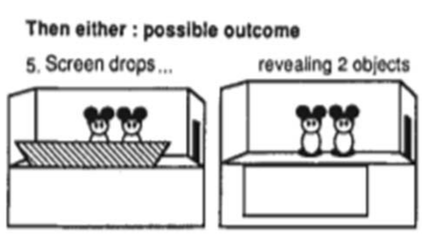


Addition and subtraction by human infants

Karen Wynn

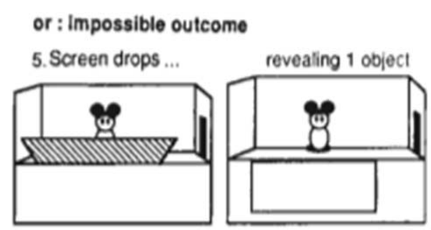
Department of Psychology, University of Arizona, Tucson, Arizona 85721, USA

Babies can Add!



NATURE · VOL 358 · 27 AUGUST 1992

Babies can Add!



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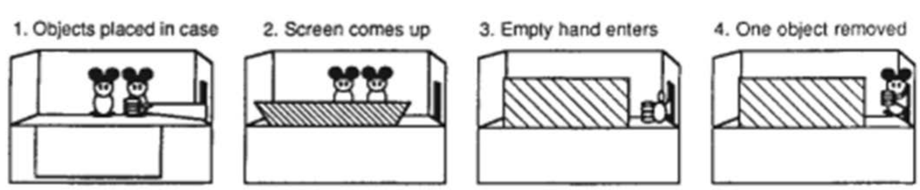
Babies can Subtract!

Sequence of events 2–1 = 1 or 2

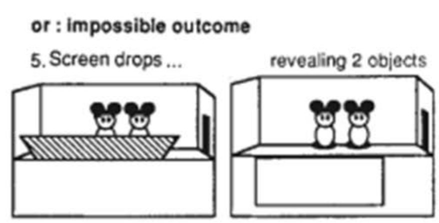


NATURE · VOL 358 · 27 AUGUST 1992

Babies can Subtract!

NATURE · VOL 358 · 27 AUGUST 1992

Babies can Subtract!



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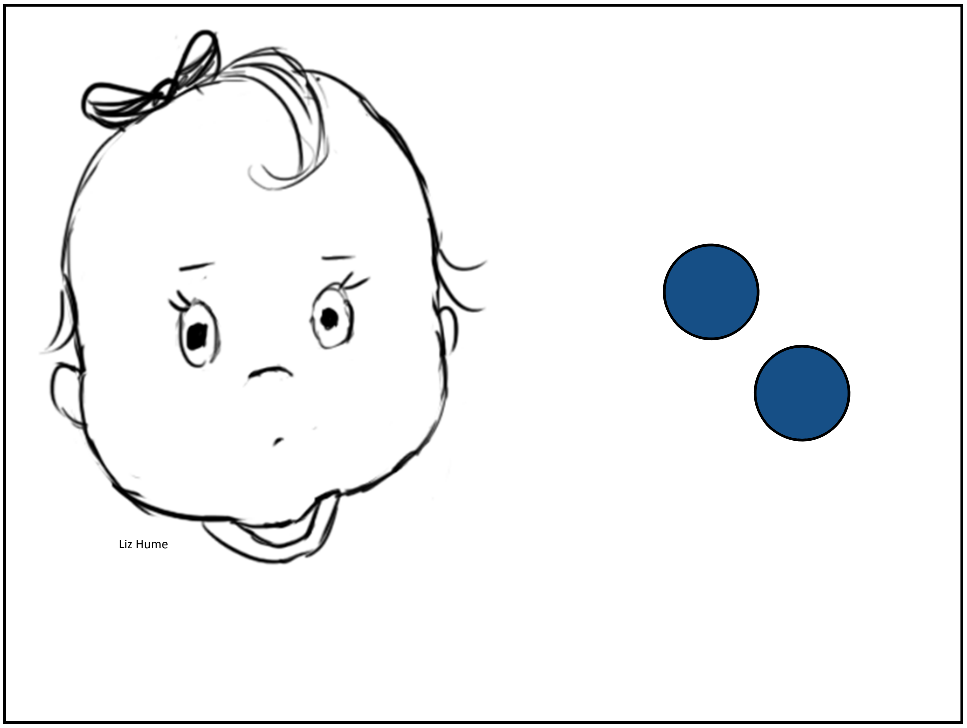
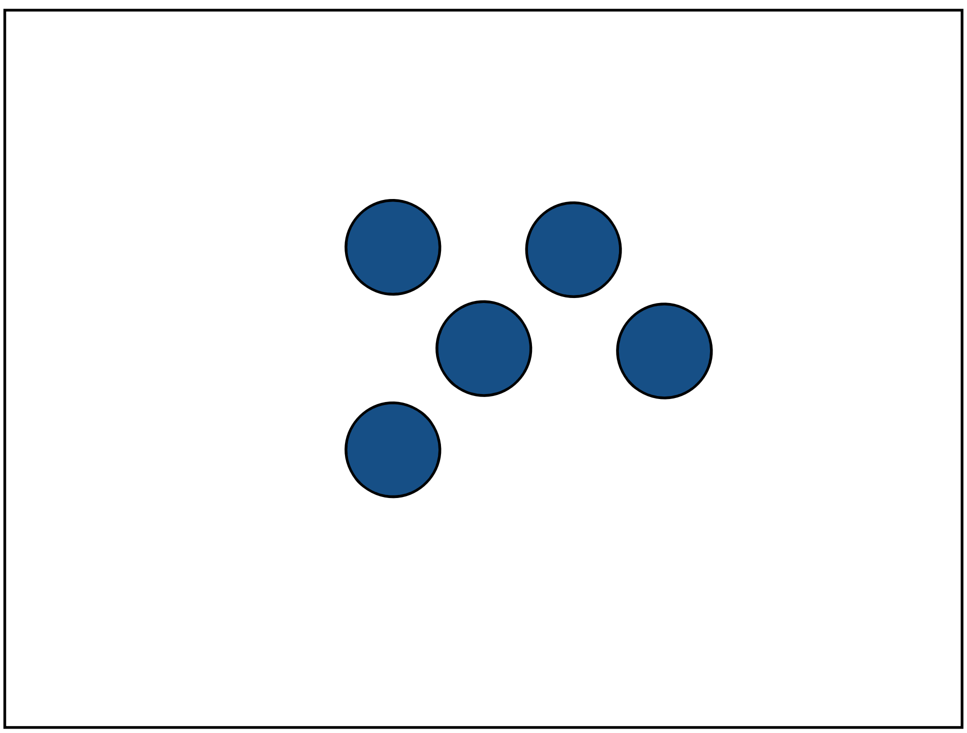
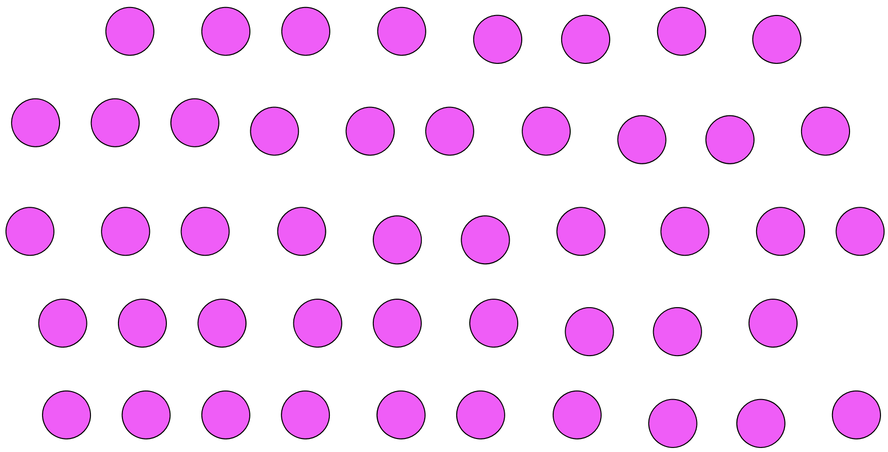
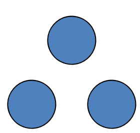
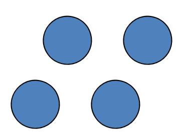

Liz Hume





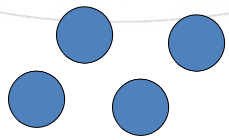
Number Sense and Instructional Choices





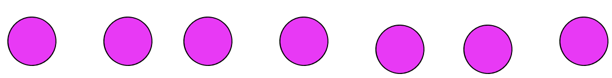
Doug Clements, Julie Sarama

Subitizing



This is a critical skill and may lay underneath early math number sense difficulties with addition and subtraction.

Number Sense and Instructional Choices

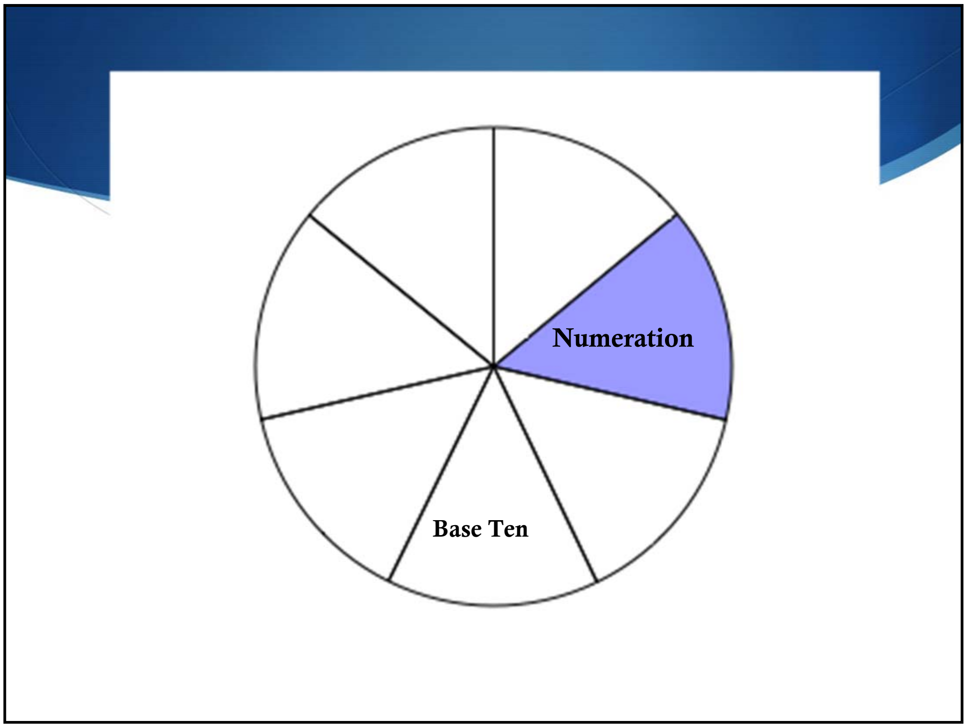


Number Sense and Instructional Choices

Concrete Reality

8 – 5 = 8

7 – 4 = 7



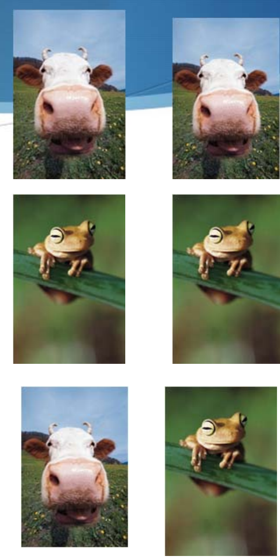
Enumerating Units!

- 3 ones and 2 ones
- 3 tens and 2 tens
- 3 tens and 2 ones
- 3/6 and 2/6
- 3/6 and 2/5

- 3X and 2X
- 3Y and 2Y
- 3X and 2Y

Connection to Categorizing

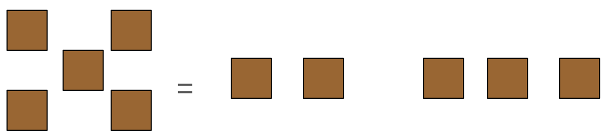
- 3 ones and 2 ones
- 3 tens and 2 tens
- 3 tens and 2 ones
- 3/6   and 2/6
- 3/6 and 2/5



"ONE IS" Video!

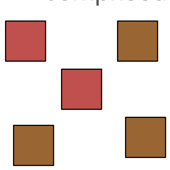
Lets start simple:

- Composition: 2+3=5
- + =
- Decomposition: 5=2+3



Trajectory of Place Value

- Pre-Part –Whole Recognizer (age 0-2) – know that objects can be combined but may not explicitly recognize that groups are comprised of smaller sets.

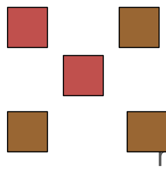


student can show you
“all the blocks but…….

cant explain 2 blue
and 3 brown make “5”

Trajectory of Place Value

- Inexact -Part –Whole Recognizer (age 3-4) – know that a whole is larger than it’s two combined parts but they may not assess the correct total.
-



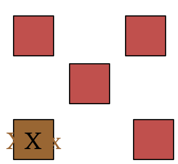
When shown 2 and 3 more objects combined

students may say I have “8” or “6” for the total (a arger number not necessarily the correct number.

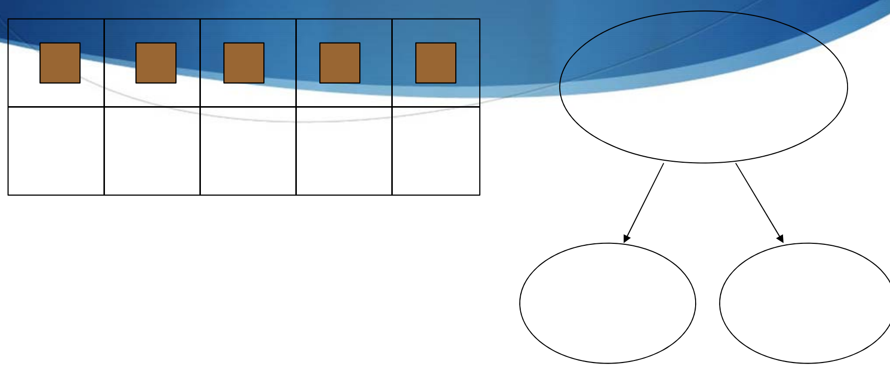
Trajectory of Place Value

- Composer to 4 then 5 (age 4-5) – knows object combinations to 4 then 5.

When students are shown 5 objects and one is taken away – they can tell you one “is missing”



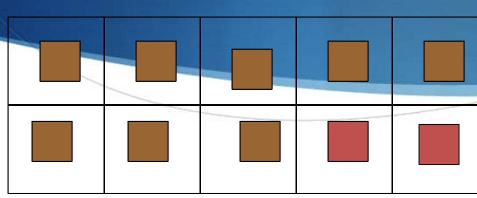
The Power of Five



Makes 10 Left over

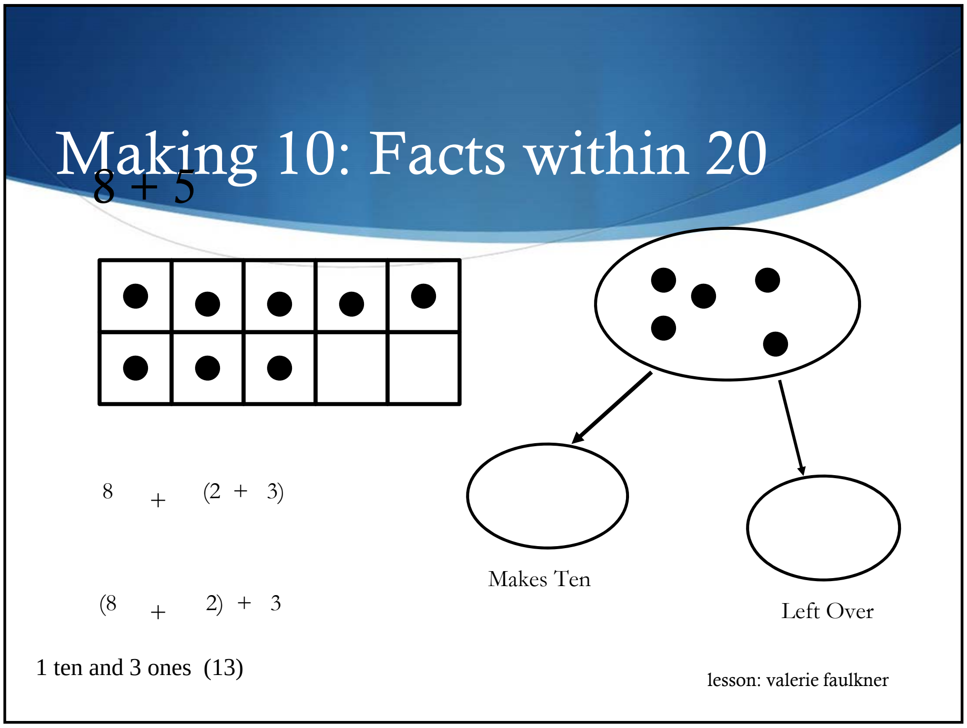
________ + ________ = ________

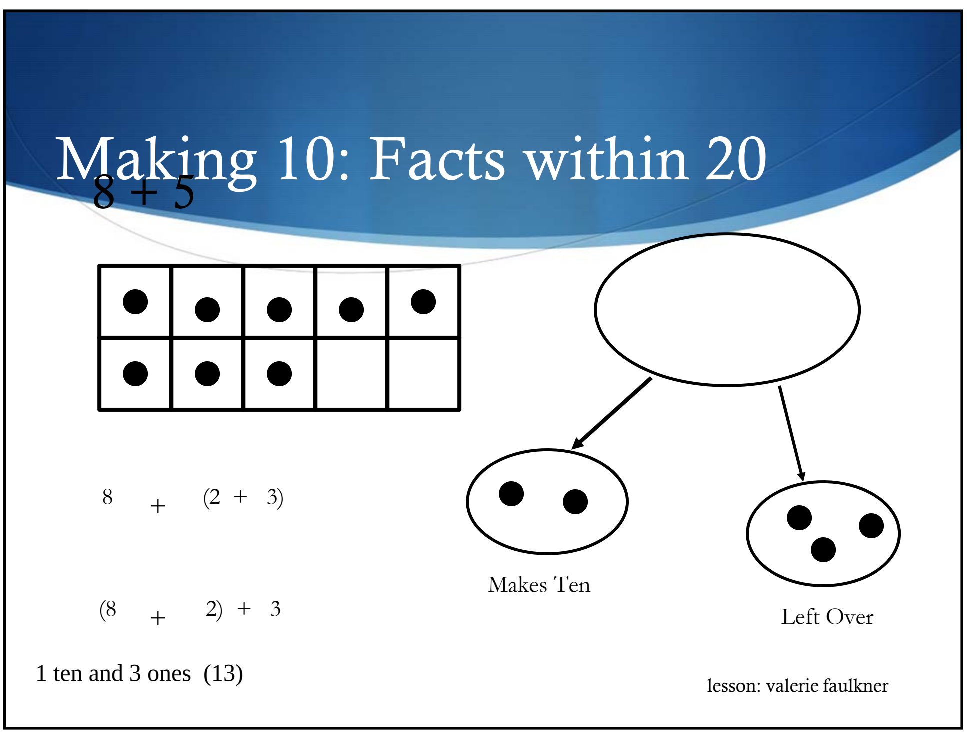
The Power of 10



__8__ + __2__ = 10

Makes 1 ten or 10 ones


Making 10: Facts within 20
8 + 5
8 + (2 + 3)
(8 + 2) + 3
1 ten and 3 ones (13)
Makes Ten
Left Over
lesson: valerie faulkner


Making 10: Facts within 20
8 + 5
8 + (2 + 3)
(8 + 2) + 3
1 ten and 3 ones (13)
Makes Ten
Left Over
lesson: valerie faulkner

Think about the power of understanding The Base-Ten Numeration System!

Fractions

What is 45% of 80?

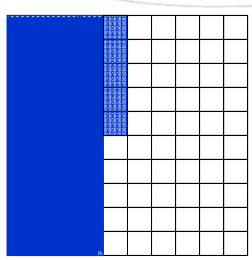
What about a child that does not know how to multiply double digit numbers or decimals?

How can we make it clear?

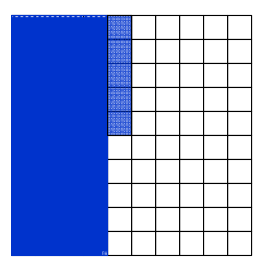
Why are pre-skills so important?

"Move it over two" or percentage and decimal relationship using the components of number sense

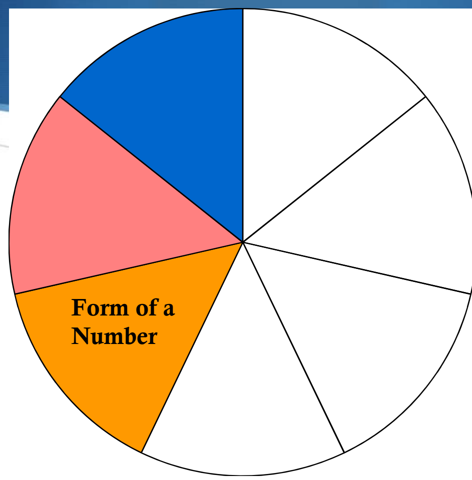
45% and .45



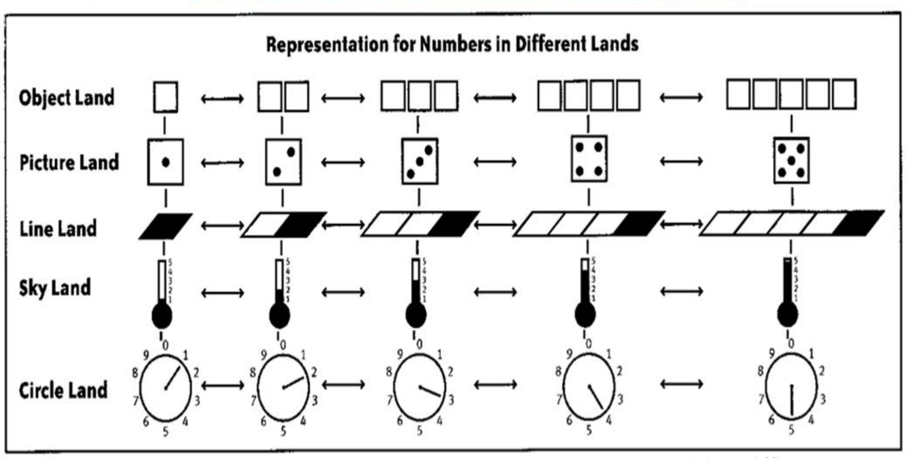
out of 100



out of 1



Different Forms of a Number



Number Worlds Griffin

Different Form of the Same Value

$$4+4 = 7+1$$

Form of Number and Properties

$$4 + (26 + 9) = (4 + 26) + 9$$

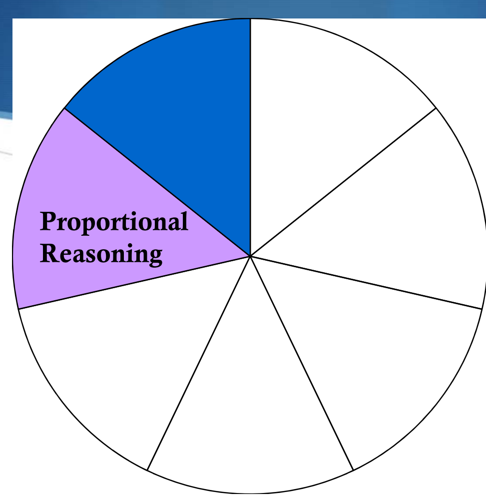
Form of Number and Properties

4 - 3

is not commutative

4 + -3

is commutative



If Sam can mow one lawn in 2 hours, how many lawns can he mow in 8 hours?

If Sam can mow one lawn in 2 hours, how many lawns can he mow in 8 hours?

$$\frac{2}{1} = \frac{8}{x}$$

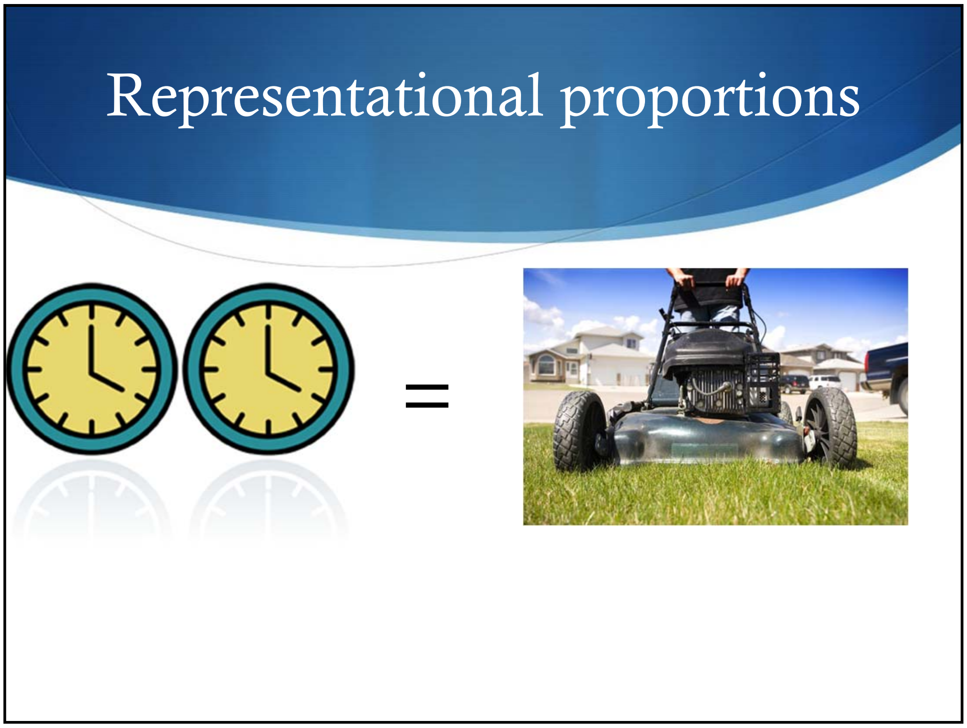
$$\frac{2}{8} = \frac{1}{x}$$

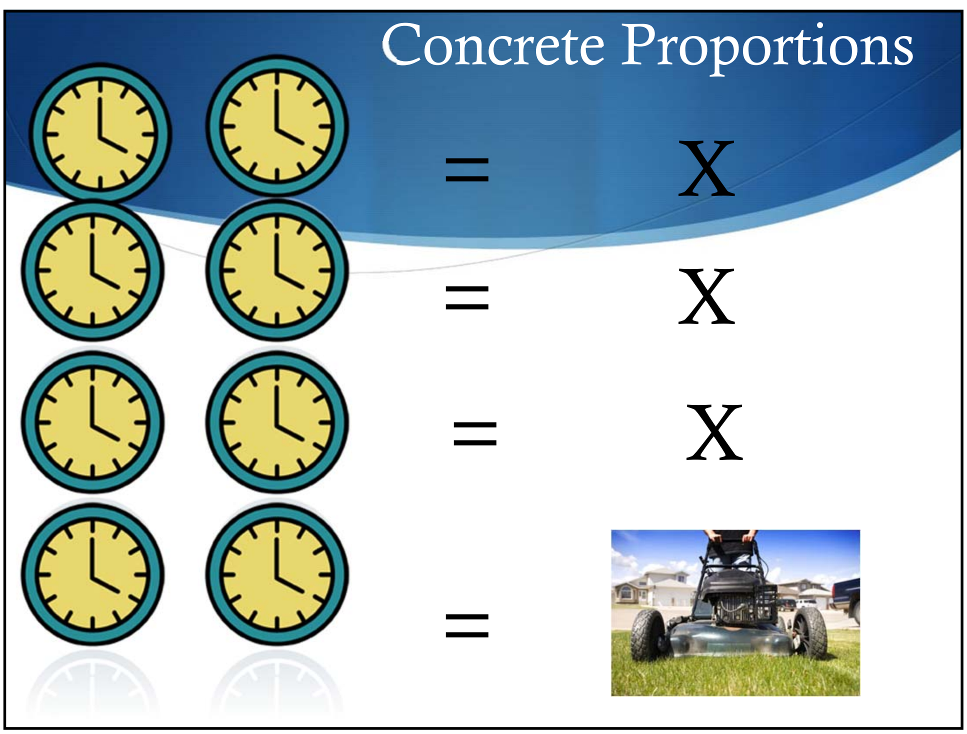
$$\frac{1}{2} = \frac{x}{8}$$

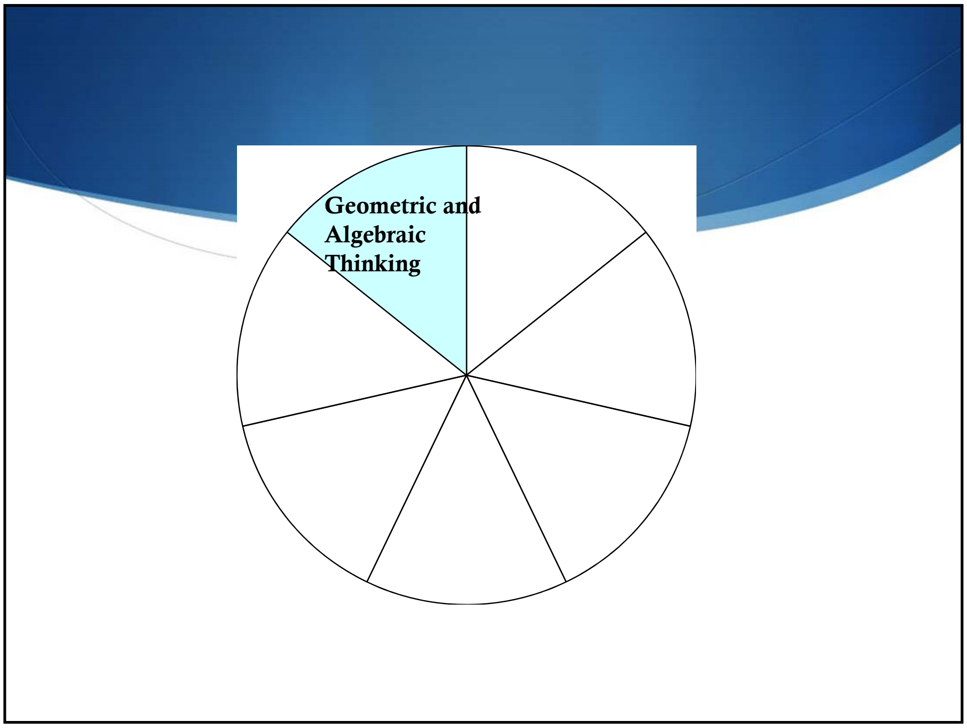
$$\frac{8}{x} = \frac{2}{1}$$

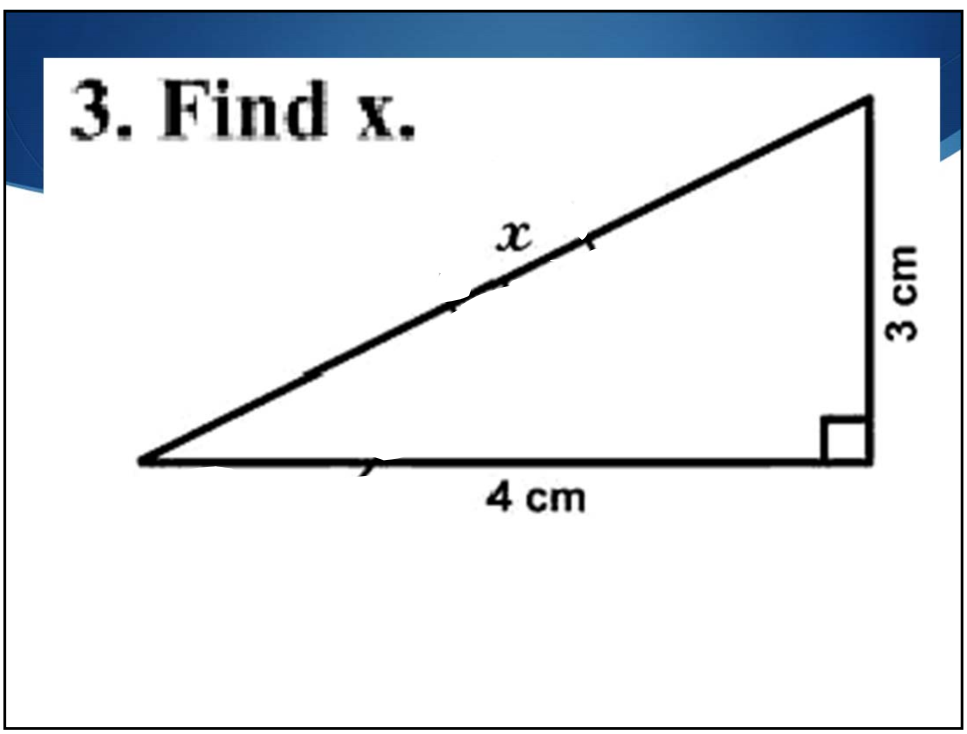
$$\frac{x}{1} = \frac{8}{2}$$

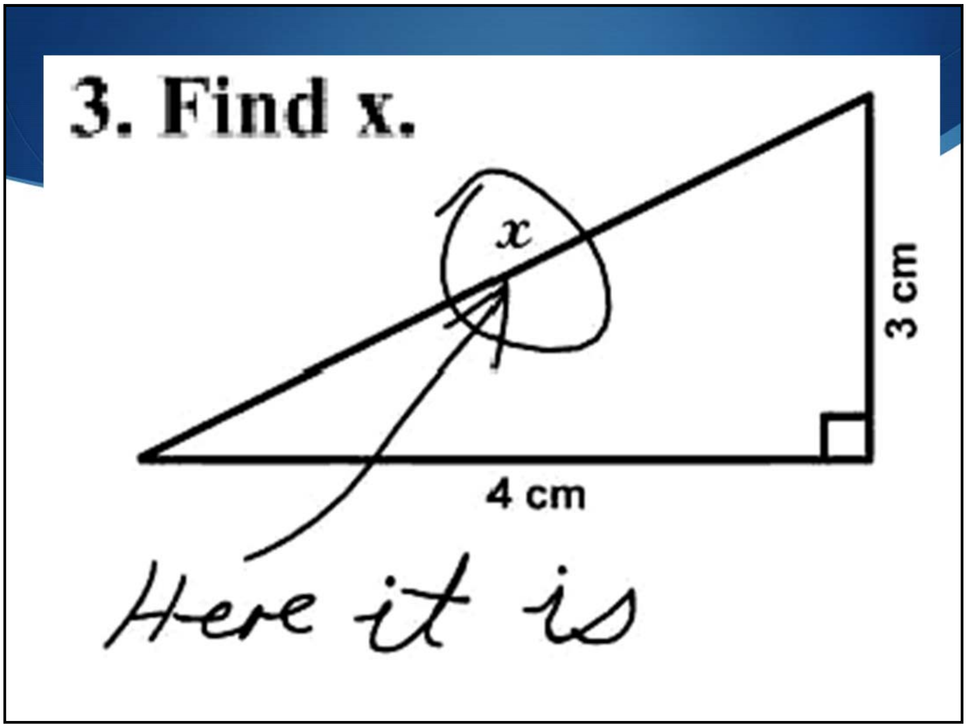
$$\frac{1}{x} = \frac{2}{8}$$


Representational proportions
=


Concrete Proportions
= X
= X
= X
=


Geometric and
Algebraic
Thinking


3. Find x.
x
3 cm
4 cm



What is a Variable?

If they are ‘variables’ why is there only ***one*** right answer?

Fixed or Variable

\+ 7 = 10

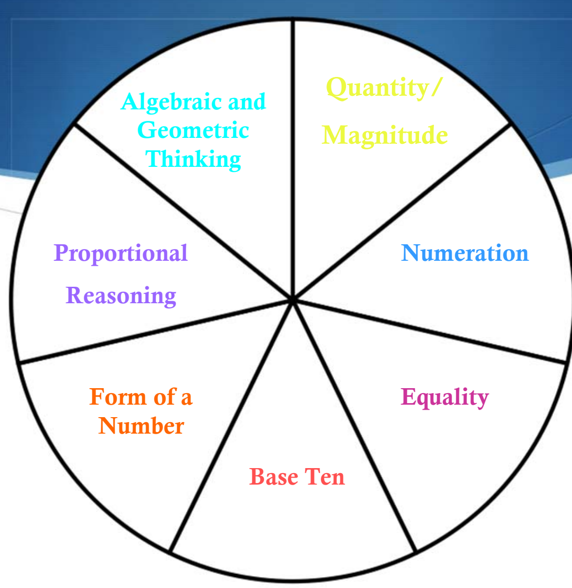
X + 7 = 10

\+ 7 =

X + 7 = Y

Know the concepts

Know how to show the concepts



Components of Number Sense (almost!)

Story One

You have a dime (1/10 of a dollar) and seven nickels (7/20 of a dollar).

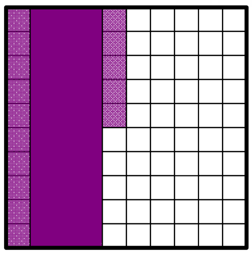
In fractional form show how much money you have all together.

Story Two

A team has two team members they made 3-point shots at the rates of 1/10 and 7/20.

How did the two of them perform as a team, what was their rate?

How Do We Represent the Money Problem Story One?



_45_ / _100__

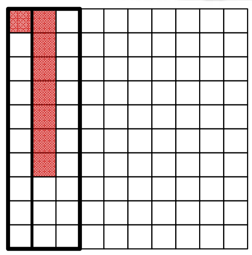
1 Dime

7 Nickels

Abstraction and Unit Size

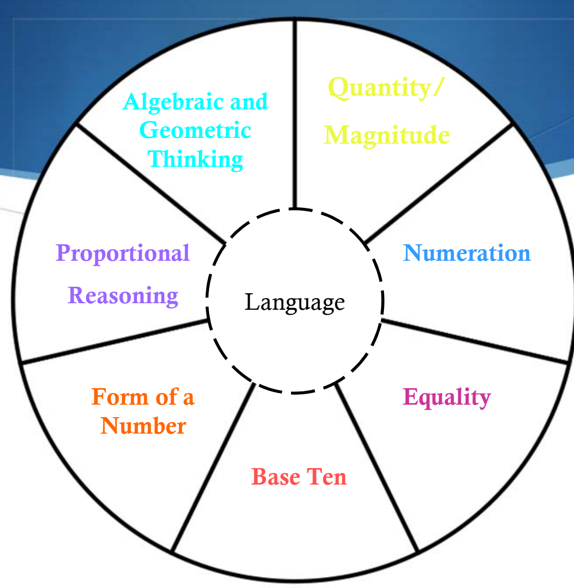
The unit size remains constant with money

How Did the Team Do Altogether Story Two?



__8_ / _30_

This story has a ***dynamic*** unit size

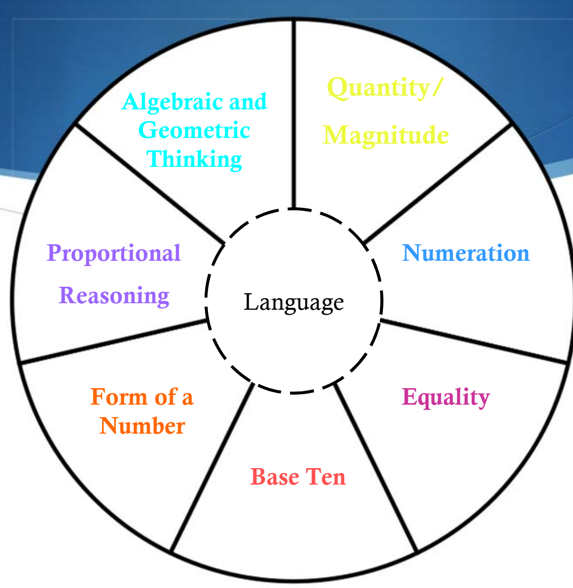


Components of Number Sense

Language and Math Language

Is math-language language?

How can we use some of what we do in reading instruction to improve our mathematics instruction?



Components of Number Sense

- Language:
  Natural, from birth…
- Reading and Writing:
  Directly connected to language
-
  Weak connection?

Language and Math Language

- Why is language different from math language? Or is it?
- Why do we teach reading so differently than we teach math?
- What do we differently?
- How do we use what we already know (language wise)

We can't teach math the way we were taught!

- Discussion in the classroom
- Multiple examples
- Make sure we master the new material before we ask children to discriminate (think of a surgeon learning a new procedure).
- Homework is made for practicing what we already know.

Precise and Consistent Language

Sponges!

They don't work…

more

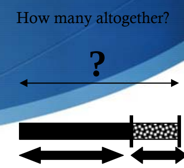
Erin has 46 comic books. She has 18 more comic books than Jason has. How many comic books does Jason have.

But is our answer really 64 which is 46 + 18?

3 Structures of Addition

Join and Part-Part Whole

- There is something, and you get more of it?
- There are two kinds, how many all together?

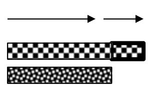


Start Unknown

- Some are given away, some are left, how many were there to start?

Compare--total unknown

- I know one amount and I have some amount more than that--how many do I have?



Addition types Adapted from Carpenter, Fennema, Franke, Levi and Empson, 1999, p. 12 in Adding it Up, NRC 2001.

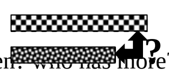
3 Structures of Subtraction

Ask yourself if a problem is a subtraction problem—
Does it fit one of these three types?:

- **The Classic** "Take away"
  (how many left?)

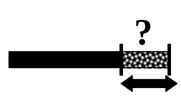
You've got some amount and "take away" from it: What's left?

- **Comparison**
  (difference between: who has more?)

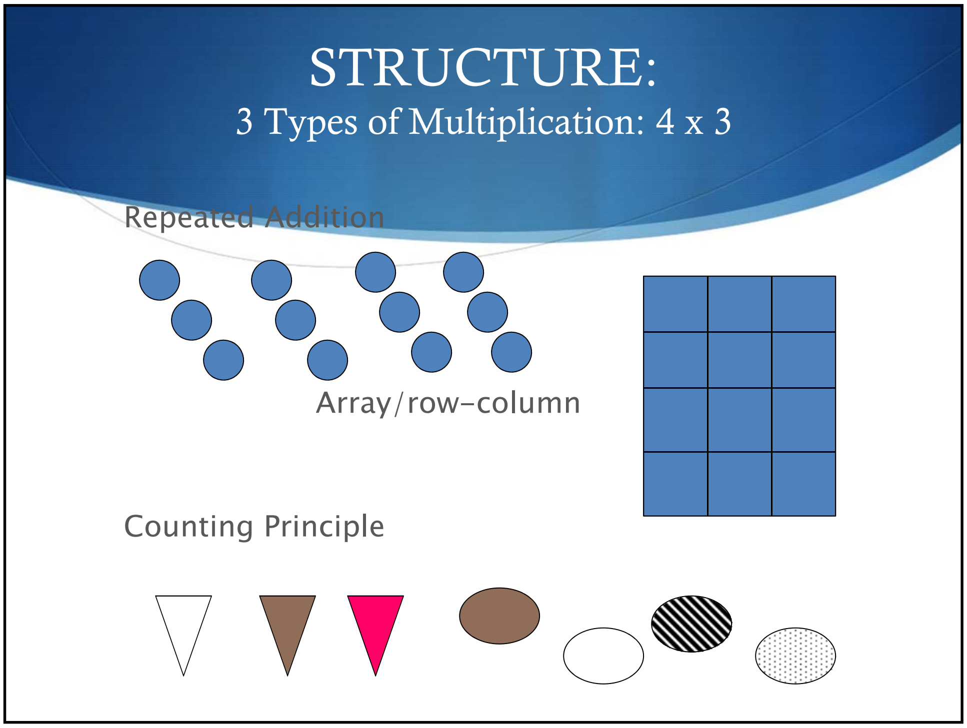


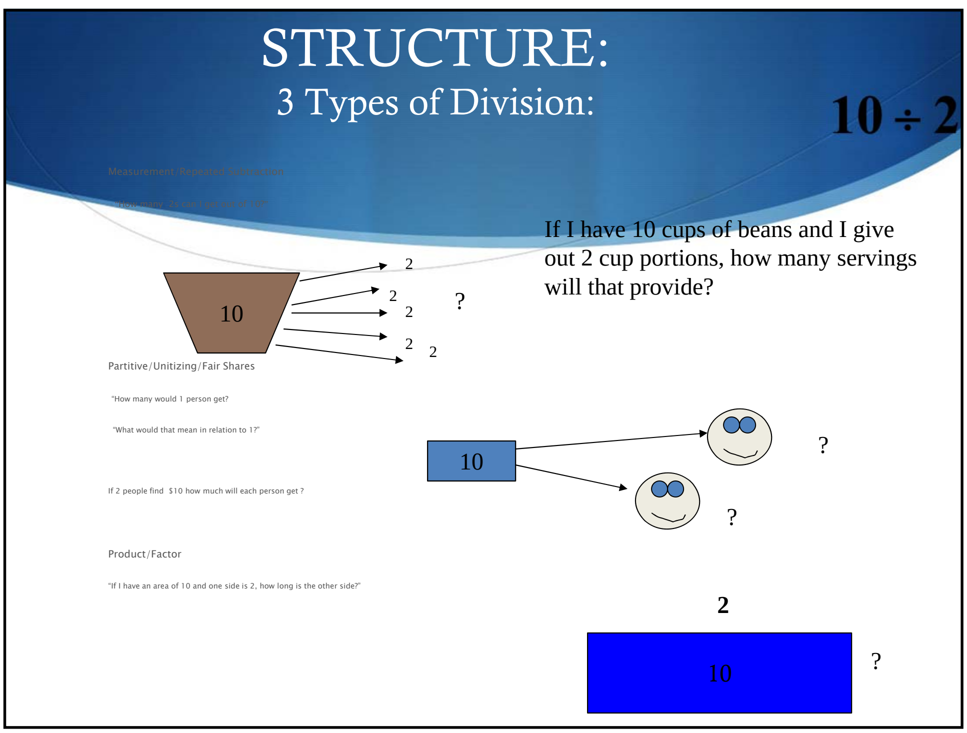
You compare to see: Who has more or less?

- **Deficit/Missing amount**
  (what's missing?)



You need some more to get where you want to be: What is the missing amount?


STRUCTURE:
3 Types of Multiplication: 4 x 3
Repeated Addition
Array/row-column
Counting Principle


STRUCTURE:
3 Types of Division:
10 ÷ 2
Measurement/Repeated Subtraction
"How many 2s can I get out of 10?"
If I have 10 cups of beans and I give out 2 cup portions, how many servings will that provide?
10
2
2
2
2
2
?
Partitive/Unitizing/Fair Shares
"How many would 1 person get?
"What would that mean in relation to 1?"
If 2 people find $10 how much will each person get ?
10
?
?
Product/Factor
"If I have an area of 10 and one side is 2, how long is the other side?"
2
10
?

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